Oh no...what we’re learning today is HARD!!??
Exploration 1-4a.
For a reminder, read #3 before starting #2

Conical Tank Problem

In a chemical plant, a conical 10' tall tank, with a 5' base radius is being emptied of liquid Benzene at a constant rate of 5 ft$^3$/min. At time $t = 0$ min., 100 ft$^3$ of Benzene is in the tank. The tank has a radius of 5 ft. and height of 10 ft.

A. Draw a picture of the situation described.
b) Volume \( V(t) \)
\[ V(t) = 100 - 5t \]

c) \[ \frac{5}{10} = \frac{r}{h} \]
\[ 10r = 5h \]
\[ r = \frac{1}{2} h \]

\[ V = \frac{1}{3} \pi \left( \frac{1}{2} h \right)^2 h \]
\[ V = \frac{1}{3} \pi \left( \frac{1}{4} h^2 \right) h \]
\[ V = \frac{1}{12} \pi h^3 \]

d) \[ R \cdot V = \frac{1}{12} \pi h^3 \cdot 12 \]
\[ 12V = \pi h^3 \]
\[ \frac{12V}{\pi} = h^3 \]
\[ \sqrt[3]{h^3} = \sqrt[3]{\frac{12V}{\pi}} \]

h = \sqrt[3]{\frac{12V}{\pi}}
e. Write an equation expressing "h" as a function of V. Use the result and the information in part a to write an equation for the composite function h(V(t)).

\[
h = \sqrt[3]{\frac{12V}{\pi}}
\]

\[
h(V(t)) = \sqrt[3]{\frac{12(100 - 5t)}{\pi}}
\]

F. At what time will the tank be completely empty?

\[
set h = 0
\]

\[
0 = \sqrt[3]{\frac{12(100 - 5t)}{\pi}}
\]

\[
0 = 1200 - 60t
\]

\[
t = \frac{1200}{60} = 20\text{ min}
\]

G. At what negative value of t was the tank completely full?

\[
set h = 10
\]

\[
10 = \sqrt[3]{\frac{12(100 - 60t)}{\pi}}
\]

\[
1000 = 1200 - 60t
\]

\[
t = \frac{1200 - 60(1941.59)}{60} = 32.36\text{ min}
\]

H. What, then, is the domain of the function h ∘ V?

\[-32.36 \leq t \leq 20\]
3. Use the following functions on the restricted domains.

\[ g(x) = 10 - 2x \quad 1 \leq x \leq 4 \]
\[ f(x) = x + 2 \quad 3 \leq x \leq 7 \]

A. Plot \( g \), \( f \), \( f \circ g \)

\[ f \circ g = 10 - 2x + 2 \quad 1.5 \leq x \leq 3.5 \]
\[ = 12 - 2x \]
\[ f(g(1.5)) \quad f(g(3.5)) \]

B. Find \( f(g(3)) \)

\[ g(3) = 4 \quad f(4) = 6 \]

C. Find the domain of \( f \circ g \).

*To do this..

Put \( g(x) \) into the domain of \( f(x) \) and solve for \( x \). Next, make sure \( x \) is in the domain of \( g \).
D. Show that \( g(f(2)) \) and \( g(f(5)) \) are both undefined but for different reasons.

\[
g(f(2)) = \text{undefined} \quad 2 \text{ is not in the domain } f(x)
\]

\[
g(f(5)) = f(5) = 7 \quad g(f(5)) = \text{undefined} \quad 7 \text{ is not in the domain of } g(x)
\]

E. Find the domain of \( g \circ f \) algebraically.

\[
f(x) = x+2 \quad g(x) \text{ domain } 1 \leq x \leq 4
\]

\[
1 \leq x+2 \leq 4 \quad \frac{-2}{2} \quad \frac{-2}{-2} \quad -1 \leq x \leq 2
\]

F. Graph \( y = g(f(x)) \)

\[
g(f(x)) = 10 - 2(x+2)
\]

\[
= 10 - 2x - 4 = 6 - 2x
\]

G. Find \( f(f(4)) \).

\[
f(4) = 6 \quad f(6) = \square
\]

H. Show \( g(g(2)) \) is undefined.

\[
g(2) = 6 \quad g(g(2)) = \text{undefined}
\]

\[
g(g(2)) = \text{not in } g(x) \text{ domain}
\]
4. Use the following functions in the restricted domains.

\[ f(x) = \sqrt{x} \quad x \geq 0 \]

\[ g(x) = x - 5 \quad \forall x \in \mathbb{R} \]

Find the domains of \( f \circ g \) and \( g \circ f \)

\[ f \circ g = x \geq 5 \]

\[ \text{Domain} \]

B. Express the volume of the Benzene in the tank in terms of \( t \).

C. Find the volume of the liquid in the tank in terms of \( h \) (Recall volume of a cone \( V = \frac{1}{3} \pi r^2 h \))

*create a proportion to start off...

D. Solve part C for \( h \).