Review Day 2 — Rational Expressions
Even More Fun and Exciting Examples

1. Rewrite each expression as a single rational expression in factored form.

a. $\frac{(x+2)}{(x+2)}\frac{2x}{(x-2)(x+1)} + \frac{x+3}{(x^2-4)} \cdot \frac{(x+1)}{(x+1)}$ $= \frac{2x^2+4x+x^2+4x+3}{(x+2)(x-2)(x+1)}$

$\frac{3x^2+8x+3}{(x+2)(x-2)(x+1)}$

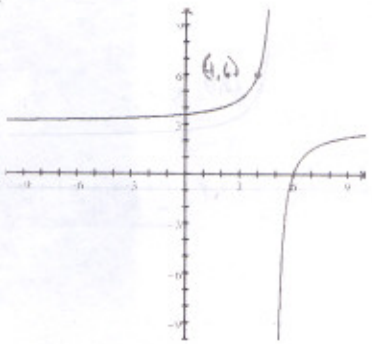
b. $\frac{x^2}{x+1} \cdot \frac{3x-6}{x^2-2x} = \frac{x^2 \cdot 3(x-2)}{(x+1)x(x-2} = \frac{3x}{x(x+1)}$

c. $\frac{\frac{x^2-5x-6}{x}}{\frac{x^2-8x+12}{x^2-1}} = \frac{(x-6)(x+1)}{x} \cdot \frac{(x+1)(x-1)}{(x-6)(x-2)} = \frac{(x+1)^2(x-1)}{x(x-2)}$

d. $\frac{(x^2+9x)}{(x^2+9x)}\frac{x}{x^2-49} - \frac{x-9}{x^2+9x} \cdot \frac{(x^2-49)}{(x^2-49)} = \frac{(x^3+9x^2)-(x^3-49x-9x^2+441)}{x(x+9)(x-7)(x+7)}$

$\frac{18x^2+49x-441}{x(x+9)(x-7)(x+7)}$

2. Write an equation for a hyperbola shown at the right that is a transformation of f(x) = 1/x. Also write this equation as a rational function.



$y = \frac{-3}{x-5} + 3$

$y = \frac{3x-18}{x-5}$

3. Rewrite $y = \frac{2x-14}{x-5}$ as a transformation of y = 1/x.

Then graph the hyperbola.

$$\begin{array}{r} 2 \\ x-5\overline{)\,2x-14} \\ -(2x-10) \\ \hline -4 \end{array}$$

$y = 2 + \frac{-4}{x-5}$

4. How can you modify $y = \frac{2x-14}{x-5}$ so that the graph is the same as the original except that it has a hole as x = -3.

$y = \frac{(2x-14)(x+3)}{(x-5)(x+3)} = \frac{2x^2+-8x-42}{x^2-2x-15}$

5. Pure gold is too soft to be used for jewelry, so gold is always mixed with other metals. 18-karat gold is 75% gold and 25% other metals. How much gold must be mixed with 5 oz of 18-karat gold to make a 22-karat (91.7%) gold mixture?

$$\frac{3.75 + x}{5 + x} = .917$$

$3.75 + x = 4.585 + .917x$

$.083x = .835$

$x = 10.0602$ oz

For each of the following rational expressions determine the x-intercepts, y-intercepts, vertical asymptotes, holes, and end behavior. Then graph each expression.

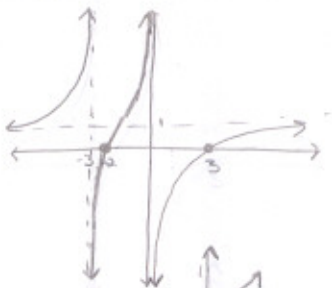
6. $f(x) = \dfrac{x^2 - x - 6}{x^2 + 3x} = \dfrac{(x-3)(x+2)}{x(x+3)}$

x-int $(3,0)$ $(-2,0)$

y-int $\emptyset$

asymp $x = 0$ $x = -3$

end behavior $y = 1$



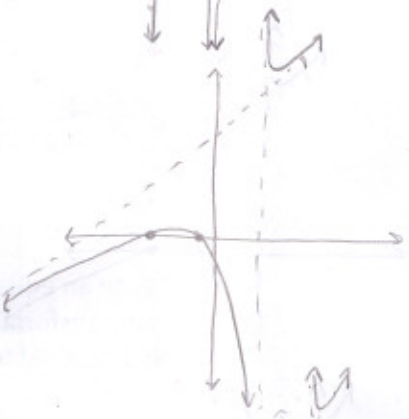
7. $g(x) = \dfrac{x^2 + 5x + 4}{x - 3} = \dfrac{(x+4)(x+1)}{x-3}$

x-int $(-4,0)(-1,0)$

y-int $(0, -\frac{4}{3})$

asymp $x = 3$

e.b. $y = x + 8$



3 | 1 5 4

3 24

1 8 | 28

8. $h(x) = \dfrac{x^3 - 8}{x - 3}$

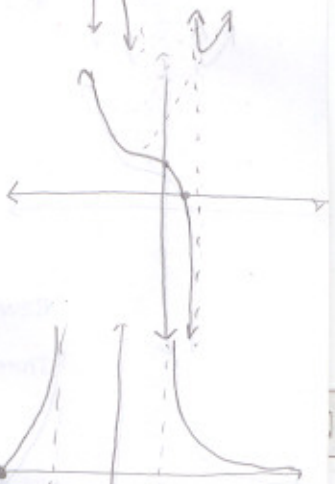
$= \dfrac{(x-2)(x^2 + 2x + 4)}{x-3}$

x-int $(2,0)$

y-int $(0, \frac{8}{3})$

asymp $x = 3$

e.b. $y = x^2 + 3x + 9$



3 | 1 0 0 -8

3 9 27

1 3 9 19

9. $k(x) = \dfrac{x^3 + 5x^2 - x - 5}{x^4 - 5x^2 + 4}$

$= \dfrac{(x+5)(x+1)(x-1)}{(x+1)(x-1)(x-2)(x+2)}$

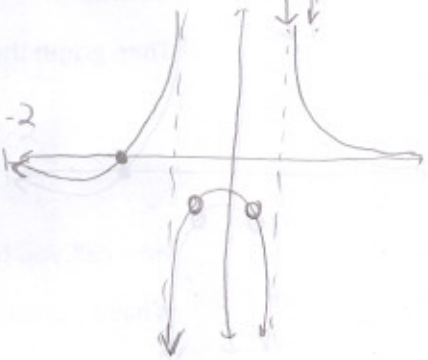
x-int $(-5,0)$

y-int $(0, -\frac{5}{4})$

asymp $x = 2$ $x = -2$

holes $x = 1$ $x = -1$

e.b. $y = 0$



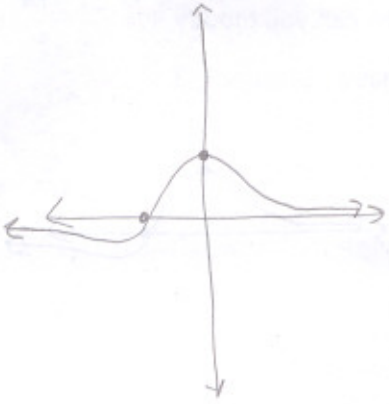
10. $m(x) = \dfrac{x+2}{x^2+1}$

x-int $(-2,0)$

y-int $(0,2)$

asymp $\emptyset$

e.b. $y = 0$



Solve the following expressions

11. $\frac{\frac{4x(x+4)}{1}\frac{1}{x+4}}{1} = \left(\frac{1}{x}+\frac{1}{4}\right)\frac{4x(x+4)}{1}$

$4x = 4(x+4) + x(x+4)$

$4x = 4x + 16 + x^2 + 4x$

$0 = x^2 + 4x + 16$

No solution

12. $\frac{(y+3)(y-3)}{1}\left(\frac{y+3}{3-y} + \frac{3y+1}{y^2-9} = \frac{1-5y}{y+3}\right) \cdot \frac{(y+3)(y-3)}{1}$

$\frac{-1(y-3)}{} \quad \frac{}{(y+3)(y-3)}$

$-1(y+3)^2 + 3y + 1 = (1-5y)(y-3)$

$-y^2 - 6y - 9 + 3y + 1 = y - 3 - 5y^2 + 15y$

$4y^2 - 18y - 5 = 0$

$(4y+1)(x-5) = 0$

$x = -\frac{1}{4}, 5$

Simplify the following expressions

13. $\frac{n - \frac{4}{n}}{1 + \frac{n}{2}} \cdot \frac{\frac{2n}{1}}{\frac{2n}{1}} = \frac{2n^2 - 8}{2n + n^2}$

$\frac{2(n+2)(n-2)}{n(n+2)} = \frac{2(n-2)}{n}$

14. $\frac{\frac{t+1}{t-1}}{\frac{2t+2}{t^2-1}} \frac{\frac{(t-1)(t+1)}{1}}{\frac{(t-1)(t+1)}{1}} = \frac{(t+1)^2}{2t+2} = \frac{(t+1)^2}{2(t+1)} = \frac{t+1}{2}$