LESSON 6.1

Exercises

1. a. 

b. 

2. 

The sum of the percentages coming out of each node must equal 1; \(0.60 + 0.40 = 1.0\) and \(0.47 + 0.53 = 1.0\).

3. 

4. a. \(A(-3,2), B(1,3), \text{ and } C(2, -2)\)

b. 

Subtract 4 units from each entry in row 2, which represents the \(y\)-coordinates.

c. 

Add 4 units to each entry in row 1, which represents the \(x\)-coordinates.

5. a. 20 girls and 25 boys. For girls, add the entries in row 1. For boys, add the entries in row 2.

b. 18 boys. The entry in row 2, column 2, is 18.

c. 13 girls batted right-handed. In matrix \([A]\), \(a_{12} = 13\) because 13 is the entry in row 1, column 2.

6. a. 

b. 

c. After 1 s: \([46 \ 194]\); after 2 s: \([51.1 \ 188.9]\).

After 1 second, 90% of the NO remains the same and 5% of the \(N_2O_3\) changes to NO, so \(NO = 40(0.90) + 200(0.05) = 46.\) After 1 second, 10% of the NO changes to \(N_2O_4\) and 90% of the \(N_2O_3\) remains the same, so \(N_2O_4 = 40(0.10) + 200(0.90) = 194.\)

Use the matrix \([46 \ 194]\) to find the amount in milliliters of NO and \(N_2O_4\) after 2 seconds.

\(NO = 46(0.90) + 194(0.05) = 51.1\)

\(N_2O_4 = 46(0.10) + 194(0.90) = 188.9\)

In matrix form: \(\begin{bmatrix} NO & N_2O_4 \end{bmatrix} = \begin{bmatrix} 46 \ 194 \end{bmatrix}\) after 1 s and \(\begin{bmatrix} NO & N_2O_4 \end{bmatrix} = \begin{bmatrix} 51.1 \ 188.9 \end{bmatrix}\) after 2 s.

7. a. 

b. 

c. After 1 yr: \([16.74 \ 8.26]\); after 2 yr: \([17.3986 \ 7.6014]\)

After 1 year:

urban = 16.74 + 9(0.10) = 16.74

To find the rural population, subtract 16.74 from the total population, 25.

rural = 25 - 16.74 = 8.26

After 2 years:

urban = 16.74(0.90) + 8.26(0.10) = 17.3986

rural = 25 - 17.3986 = 7.6014

In matrix form: after 1 yr, \([urban \ rural]\) = \([16.74 \ 8.26]\); after 2 yr, \([urban \ rural]\) = \([17.3986 \ 7.6014]\).

8. a. There were 50 history books sold at the branch bookstore. In matrix \([A]\), \(a_{22} = 50\) because 50 is the entry in row 3, column 2, where row 3 represents history and column 2 represents the branch bookstore.

b. There were 65 science books sold at the main bookstore. In matrix \([A]\), \(a_{21} = 65\) because 65 is the entry in row 2, column 1, where row 2 represents science and column 1 represents the main bookstore.

c. This week there were 3 more math books sold at the main bookstore and 8 more math books sold at the branch bookstore. The entries in row 1, column 1, for matrices \([A]\) and \([B]\) represent the
number of math books sold at the main bookstore this week and last week, respectively. The difference between \( a_{11} = 83 \) and \( b_{11} = 80 \) is 3, so 3 more math books were sold at the main bookstore this week. Similarly, the entries in row 1, column 2, for matrices \([A]\) and \([B]\) represent the number of math books sold at the branch bookstore this week and last week, respectively. The difference between \( a_{12} = 33 \) and \( b_{12} = 25 \) is 8, so 8 more math books were sold at the branch bookstore this week.

**d.** Because the rows and columns have the same meaning in both matrices, you can add the corresponding entries in matrices \([A]\) and \([B]\).

\[
\begin{bmatrix}
83 + 80 & 33 + 25 \\
65 + 65 & 20 + 15 \\
98 + 105 & 50 + 55
\end{bmatrix} = \begin{bmatrix}
163 & 58 \\
130 & 35 \\
203 & 105
\end{bmatrix}
\]

**9. a.**

**b.**

\[
\begin{bmatrix}
.62 & .20 & .18 \\
.35 & .45 & .20 \\
.12 & .32 & .56
\end{bmatrix}
\]

**c.** The sum of each row is 1. The percentages should add to 100%.

**10. a.**

**b.**

\[
\begin{bmatrix}
.65 & .20 & .15 \\
.25 & .55 & .20 \\
.40 & .30 & .30
\end{bmatrix}
\]

**c.** The sum of each row is 1. The percentages should add to 100%.

**d.** 47 trucks in Bay County, 33 trucks in Sage County, and 20 trucks in Thyme County.

Bay: \(45(.65) + 30(.25) + 25(.40) = 46.75 \approx 47\)

Sage: \(45(.20) + 30(.55) + 25(.30) = 33\)

Thyme: \(45(.15) + 30(.20) + 25(.30) = 20.25 \approx 20\)

**11. a.** \(5 \times 5\); matrix \([M]\) has five rows and five columns.

b. \(m_{32} = 1\) because 1 is the entry in row 3, column 2. There is one round-trip flight between City C and City B.

c. City A has the most flights. From the route map, more paths have A as an endpoint than any other city. From the matrix, the sum of row 1 (or column 1) is greater than the sum of any other row (or column).

d. Start with four points, J, K, L, and M. Then use the numbers in the matrix to tell you how many paths to draw between endpoints. For example, \(n_{13} = 2\) tells you to draw 2 paths between J and L.

**12.** \((\frac{37}{9}, -\frac{10}{9})\). To solve by substitution, solve the second equation for \(x: x = 3 - y\). Substitute \((3 - y)\) into the first equation and solve for \(y\).

\[
5(3 - y) - 4y = 25
\]

\[
15 - 5y - 4y = 25
\]

\[
-9y = 10
\]

\[
y = -\frac{10}{9}
\]

Substitute \(-\frac{10}{9}\) for \(y\) in either equation and solve for \(x\): \(x = 3 - \left(-\frac{10}{9}\right) = \frac{37}{9}\).

To solve by elimination, multiply the second equation by 4 and add the two equations. Once \(y\) is eliminated, solve for \(x\). Then substitute the value of \(x\) into either equation and solve for \(y\).

**13.** \(7.4p + 4.7s = 100\)

**14.** \(3y = 12 - 2x\), so \(y = 4 - \frac{2}{3}x\). The \(x\)-intercept is 4 and the slope is \(-\frac{2}{3}\).
15. a. Let \( x \) represent the year, and let \( y \) represent the number of subscribers.

b. \( y = 1231000(1.44)^{x-1987} \). Use the point-ratio method with the points (1987, 1231000) and (1998, 69209000). Substitute the coordinates of the points into point-ratio form, \( y = y_1 \cdot b^{x-x_1} \).

\[ y = 1231000b^{x-1987} \text{ and } y = 69209000b^{x-1998} \]

Use substitution to combine the two equations and solve for \( b \).

\[ 1231000b^{x-1987} = 69209000b^{x-1998} \]

\[ \frac{b^{x-1987}}{b^{x-1998}} = \frac{69209000}{1231000} \]

\[ b^{11} = \frac{69209000}{1231000} \]

\[ b = \left(\frac{69209000}{1231000}\right)^{1/11} \approx 1.44 \]

Substitute 1.44 for \( b \) in either of the two original equations. The exponential equation is \( y = 1231000(1.44)^{x-1987} \).

c. About 420,782,749 subscribers. Substitute 2003 for \( x \) in the equation from 15b:

\[ y = 1231000(1.44)^{2003-1987} \approx 420,782,749. \]

Therefore there are about 420,782,749 subscribers in 2003. This is not a realistic prediction because it far exceeds the 2003 population of the United States. (The population was about 288 million in 2002.) It is unlikely that there will ever be more than one cellular phone for every person in the country. The number of cell phones cannot possibly continue to increase at a rate of 44% a year. Because the number of cell phones will level off rather than continue to increase exponentially, an exponential model is not appropriate.

d. \( y = (0.00002)(10)^{x/20} \). To find the inverse function, switch \( x \) and \( y \) and then solve for \( y \).

\[ x = 20 \log \left(\frac{y}{0.00002}\right) \]

\[ 10^x = \left(\frac{y}{0.00002}\right)^{20} \]

\[ (10^x)^{1/20} = \frac{y}{0.00002} \]

\[ y = (0.00002)(10)^{x/20} \]

Therefore the inverse function is \( y = (0.00002)(10)^{x/20} \), where \( x \) represents the intensity in decibels and \( y \) represents pressure in Pascals.

d. 0.63246 Pa. Substitute 90 for \( x \) in the inverse function: \( y = (0.00002)(10)^{90/20} \approx 0.63246 \) Pa.

**Extension**

Answers will vary.

**Lesson 6.2**

**Exercises**

1. 197 students will choose ice cream and 43 will choose frozen yogurt.

\[ [203.35 \quad 36.65] \begin{bmatrix} .95 & .05 \\ .10 & .90 \end{bmatrix} = \]

\[ [196.8475 \quad 43.1525] \]

Multiply row 1 by column 1:

203.35(.95) + 36.65(.10) = 196.8475 \approx 197

Multiply row 1 by column 2:

203.35(.05) + 36.65(.90) = 43.1525 \approx 43

2. a. \( x = 7, y = 54 \)

\[ [13 \quad 23] + [-6 \quad 31] = [13 + (-6) \quad 23 + 31] = [7 \quad 54] \]

b. \( c_{11} = 0.815, c_{12} = 0.185, c_{21} = 0.0925, c_{22} = 0.9075 \)

\[ \begin{bmatrix} .90 & .10 \\ .05 & .95 \end{bmatrix} \begin{bmatrix} .90 & .10 \\ .05 & .95 \end{bmatrix} = \]

\[ \begin{bmatrix} .90(.90) + .10(.05) & .90(.10) + .10(.95) \\ .05(.90) + .95(.05) & .05(.10) + .95(.95) \end{bmatrix} = \]

\[ \begin{bmatrix} .815 & .185 \\ .0925 & .9075 \end{bmatrix} \]
c. \( a = 15.6, b = -10.8, c = 10.7, d = 42.2 \)
\[
\begin{bmatrix}
18 & -23 \\
5.4 & 32.2
\end{bmatrix} + \begin{bmatrix}
-2.4 & 12.2 \\
5.3 & 10
\end{bmatrix} = \\
\begin{bmatrix}
18 + (-2.4) & -23 + 12.2 \\
5.4 + 5.3 & 32.2 + 10
\end{bmatrix} = \\
\begin{bmatrix}
15.6 & -10.8 \\
10.7 & 42.2
\end{bmatrix}
\]

d. \( m_{11} = 180, m_{13} = -230, m_{21} = 54, m_{23} = 322 \)
\[
\begin{align*}
10 \begin{bmatrix}
18 & -23 \\
5.4 & 32.2
\end{bmatrix} &= \begin{bmatrix}
10 \times 18 & 10 \times (-23) \\
10 \times 5.4 & 10 \times 32.2
\end{bmatrix} \\
&= \begin{bmatrix}
180 & -230 \\
54 & 322
\end{bmatrix}
\end{align*}
\]
e. \( a = -5, b = 57, c = 44.5, d = 78 \)
\[
\begin{bmatrix}
7 & -4 \\
18 & 28
\end{bmatrix} + 5 \begin{bmatrix}
-2.4 & 12.2 \\
5.3 & 10
\end{bmatrix} = \\
\begin{bmatrix}
7 -4 & 7 + (-2.4) \\
18 & 28 + 5.3
\end{bmatrix} = \\
\begin{bmatrix}
4.6 & 1 \frac{3}{2} \\
23.5 & 33
\end{bmatrix}
\]
f. Not possible; the dimensions aren't the same. The entries in the first matrix do not match up with the entries in the second matrix.

4. \( \begin{bmatrix}
3 & -4 & 2.5 \\
-2 & 6 & 4
\end{bmatrix} \).
You can solve this matrix equation similar to solving an algebraic equation.
First subtract \( \begin{bmatrix}
8 & -5 & 4.5 \\
-6 & 9.5 & 5
\end{bmatrix} \) from both sides.
\[
-B = \begin{bmatrix}
-5 & -1 & 2 \\
-4 & 3.5 & 1
\end{bmatrix} - \begin{bmatrix}
8 & -5 & 4.5 \\
-6 & 9.5 & 5
\end{bmatrix} = \\
\begin{bmatrix}
-3 & 4 & -2.5 \\
2 & -6 & -4
\end{bmatrix}
\]
Then multiply both sides by the scalar \(-1\).
\[
B = -1 \begin{bmatrix}
-3 & 4 & -2.5 \\
2 & -6 & -4
\end{bmatrix} = \\
\begin{bmatrix}
3 & -4 & 2.5 \\
-2 & 6 & 4
\end{bmatrix}
\]

5. a. The coordinates of the vertices are organized in columns.

\[
\begin{bmatrix}
1 & 2 \\
3 & -2 \\
0 & 1
\end{bmatrix}
\]

b. \( \begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix} \)
\[
\begin{bmatrix}
-1(-3) + 0(2) & -1(1) + 0(3) \\
0(-3) + 1(2) & 0(1) + 1(3)
\end{bmatrix} = \\
\begin{bmatrix}
3 & -1 \\
2 & 3
\end{bmatrix}
\]
c. \( \begin{bmatrix}
5 & 2 \\
-17 & 8
\end{bmatrix} \)
\[
\begin{bmatrix}
5(3) + 2(-2) & 5(7) + 2(1)
\end{bmatrix} = \\
\begin{bmatrix}
13 & 29
\end{bmatrix}
\]
d. Not possible; the inside dimensions do not match. The four rows of the first matrix do not match up with the two columns of the second matrix.

\[
\begin{bmatrix}
3 & -4 \\
6 & 1
\end{bmatrix} - \begin{bmatrix}
-1 & 7 \\
-8 & 3
\end{bmatrix} = \\
\begin{bmatrix}
3 - (-1) & 6 - 7 \\
-4 - (-8) & 1 - 3
\end{bmatrix} = \\
\begin{bmatrix}
4 & -1 \\
4 & -2
\end{bmatrix}
\]
6. \( [\mathbf{A}] = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \); \( [\mathbf{C}] = \begin{bmatrix} -3 & 1 & 2 \\ -2 & -3 & 2 \end{bmatrix} \)

To find matrix \([\mathbf{C}]\), change the signs of the \(y\)-value of the original matrix, \( [\mathbf{A}] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \), because this matrix will keep the \(x\)-values the same but change the sign of the \(y\)-values.

7. a. [Image 0x0 to 613x790]

b. \([4800 \quad 4200]\)  
c. \([.72 \quad .28] \quad .12 \quad .88\)

d. \([4800 \quad 4200]\) \(\begin{bmatrix} .72 & .28 \\ .12 & .88 \end{bmatrix} = [4800(.72) + 4200(.12)] = [3960 \quad 5040]

c. \([3960 \quad 5040]\) \(\begin{bmatrix} .72 & .28 \\ .12 & .88 \end{bmatrix} = [3960(.72) + 5040(.12)] = [3456 \quad 5544]

8. a. \( [\mathbf{A}][\mathbf{B}] = \begin{bmatrix} 6 & 2 \\ -3 & -6 \end{bmatrix} \); \( [\mathbf{B}][\mathbf{A}] = \begin{bmatrix} 2 & 13 \\ 2 & -2 \end{bmatrix} \)

They are not the same.

b. \( [\mathbf{A}][\mathbf{C}] = \begin{bmatrix} -7 & 21 & 12 \\ 1 & 2 & 4 \end{bmatrix} \). You cannot multiply \([\mathbf{C}][\mathbf{A}]\) because the inside dimensions are not the same. In order for both \([\mathbf{A}][\mathbf{C}]\) and \([\mathbf{C}][\mathbf{A}]\) to exist, both have to be square matrices.

c. \( [\mathbf{A}][\mathbf{D}] = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} \); \( [\mathbf{D}][\mathbf{A}] = \begin{bmatrix} 2 & 13 \\ -1 & 1 \end{bmatrix} \).

They are the same and equal to \([\mathbf{A}]\). Multiplying by \([\mathbf{D}]\) does not change \([\mathbf{A}]\).

d. No, matrix multiplication is generally not commutative. Order does matter. In 8c, multiplying by matrix \([\mathbf{D}]\) is an exception.

9. a. \( a = 3, \ b = 4 \)

\[ \begin{bmatrix} 2 & a \\ b & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 19 \\ 17 \end{bmatrix} \]

\( 2(3) + a(3) = 19 \), so \( 3a = 9 \), and \( a = 3 \)

\( b(5) + (-1)(3) = 17 \), so \( 5b = 20 \), and \( b = 4 \)

b. \( a = 7, \ b = 4 \)

\[ \begin{bmatrix} a & -2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ -b \end{bmatrix} = \begin{bmatrix} -29 \\ -5 \end{bmatrix} \]

\( a(-3) + (-2)(b) = -29 \), so \(-3a - 2b = -29 \)

\( 3(-3) + 1(b) = -5 \), so \( b = -5 + 9 \), and \( b = 4 \)

Substitute 4 for \( b \) in \(-3a - 2b = -29\) and solve for \( a \).

\(-3a - 2(4) = -29 \), so \(-3a = -21 \), and \( a = 7 \)

10. [160 \quad 80]. An equilibrium is reached because 10% of 80 students is the same as 5% of 160 students.

11. The probability that the spider is in room 1 after four room changes is .375. The long-run probabilities for rooms 1, 2, and 3 are [.3 \ .5 \ .5].

Use this transition matrix, where rows and columns represent room 1, room 2, and room 3, in order.

\[ \begin{bmatrix} 0 & .5 & .5 \\ .5 & 0 & .5 \\ .5 & .5 & 0 \end{bmatrix} \]

After one room change:

\[ \begin{bmatrix} 1 & 0 & 0 \\ 0 & .5 & .5 \\ 0 & .5 & .5 \end{bmatrix} = \begin{bmatrix} 0 & .5 & .5 \\ 0 & .5 & .5 \\ 0 & .5 & .5 \end{bmatrix} \]

After two room changes:

\[ \begin{bmatrix} 0 & .5 & .5 \\ .5 & 0 & .5 \\ .5 & .5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & .5 & .5 \\ .5 & 0 & .5 \\ .5 & .5 & 0 \end{bmatrix} \]

After three room changes:

\[ \begin{bmatrix} .5 & .25 & .25 \\ .5 & 0 & .5 \\ .5 & .5 & 0 \end{bmatrix} = \begin{bmatrix} .25 & .375 & .375 \\ .25 & .375 & .375 \\ .375 & .375 & .375 \end{bmatrix} \]

After four room changes:
Multiplying the initial condition by the transition matrix four times, or alternatively multiplying by the transition matrix raised to the power of 4, gives $[.375 \quad .3125 \quad .3125]$. Therefore the probability that the spider is in room 1 after four room changes is .375.

By raising the matrix to a higher power, such as 40, you will see that the long-run probabilities are $[.3 \quad .3 \quad .3]$.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ .3333 & .3333 & .3333 & .3333 \\ .3 & .3 & .3 & .3 \\ .3 & .3 & .3 & .3 \end{bmatrix} \to \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

12. a. $\begin{bmatrix} .5 & .45 & .05 \\ .25 & .5 & .25 \\ .3 & .3 & .4 \end{bmatrix}$

b. After one generation: [32% 45.75% 22.25%]

After two generations:
[34.1125% 43.95% 21.9375%]

After three generations:
[34.625% 43.90625% 21.46875%]

In the long run:
[34.7403125% 44.015625% 21.23515625%]

For each generation, multiply the matrix representing the percentages in each category by the transition matrix.

After one generation:
$\begin{bmatrix} .5 & .45 & .05 \\ .25 & .5 & .25 \\ .3 & .3 & .4 \end{bmatrix} \begin{bmatrix} .25 & .60 & .15 \\ .25 & .5 & .25 \\ .3 & .3 & .4 \end{bmatrix} = \begin{bmatrix} .32 & .4575 & .225 \end{bmatrix}$

After two generations:
$\begin{bmatrix} .32 & .4575 & .225 \end{bmatrix} \begin{bmatrix} .5 & .45 & .05 \\ .25 & .5 & .25 \\ .3 & .3 & .4 \end{bmatrix} = \begin{bmatrix} .341125 & .4395 & .219375 \end{bmatrix}$

After three generations:
$\begin{bmatrix} .341125 & .4395 & .219375 \end{bmatrix} \begin{bmatrix} .5 & .45 & .05 \\ .25 & .5 & .25 \\ .3 & .3 & .4 \end{bmatrix} = \begin{bmatrix} .34625 & .4390625 & .2146875 \end{bmatrix}$

By raising the matrix to a higher power, such as 15, you will see that the long-run percentages are $[.3474042232 44.015625 21.23515625]$.

13. a. $\begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

b. The first and last UPCs are valid. Multiply the UPCs matrix by the matrix in 13a.

$\begin{bmatrix} 0 & 3 & 6 & 2 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 5 \\ 0 & 7 & 6 & 1 & 0 & 7 & 0 & 2 & 2 & 3 & 6 & - \\ 0 & 7 & 4 & 2 & 2 & 0 & 0 & 0 & 2 & 9 & 1 & 8 \\ 0 & 8 & 5 & 3 & 9 & 1 & 7 & 8 & 6 & 2 & 2 & 1 \end{bmatrix}$

The first and last UPC codes resulted in multiples of 10, so they are valid.

c. For the second code, the check digit should be 7.
For the third code, the check digit should be 5.
For the second code, the check digit needs to be raised by 1 for a product of 60, so it should be 6 + 1, or 7. For the third code, the check digit needs to be reduced by 3 for a product of 50, so it should be 8 - 3, or 5.

14. a. i. Consistent and independent because there is exactly one solution.

The first and last UPC codes resulted in multiples of 10, so they are valid.

c. For the second code, the check digit should be 7.
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For the second code, the check digit needs to be raised by 1 for a product of 60, so it should be 6 + 1, or 7. For the third code, the check digit needs to be reduced by 3 for a product of 50, so it should be 8 - 3, or 5.
ii. Inconsistent because there are no solutions. The lines are parallel.

[−10, 10, 1, −10, 10, 1]

iii. Inconsistent because there are no solutions. The lines are parallel.

[−10, 10, 1, −10, 10, 1]

iv. Consistent and independent because there is exactly one solution.

[−10, 10, 1, −10, 10, 1]

v. Inconsistent because there are no solutions. The lines are parallel.

[−10, 10, 1, −10, 10, 1]

vi. Consistent and dependent because there are infinitely many solutions. The two lines are the same.

[−10, 10, 1, −10, 10, 1]

vii. Consistent and dependent because there are infinitely many solutions. The two lines are the same.

[−10, 10, 1, −10, 10, 1]

viii. Consistent and independent because there is exactly one solution.

[−10, 10, 1, −10, 10, 1]

ix. Consistent and dependent because there are infinitely many solutions. The two lines are the same.

[−10, 10, 1, −10, 10, 1]

b. The graphs of inconsistent linear systems are parallel lines.

c. When you try to solve an inconsistent system, you reach a false statement, such as 0 = 7.

ii. By substitution, \( \frac{3}{2}x - 4 = 0.75x + 3 \), so \( 0 = 7 \), which is false.

iii. Multiply the first equation by 12 and the second equation by −4, and use elimination.

\[
\begin{align*}
4.8x + 7.2y &= 10.8 \\
-4.8x - 7.2y &= -18.8 \\
0 &= -10
\end{align*}
\]

The statement \( 0 = -10 \) is false.

v. By substitution, \( 1.2x + 3 = 1.2x - 1 \), so \( 0 = -4 \), which is false.

d. You can recognize an inconsistent linear system without graphing it because the equations have the same slope but different y-intercepts.

e. The graphs of consistent and dependent linear systems are the same line.
f. When you try to solve a consistent and dependent system, you get a true but useless statement, such as \(0 = 0\).

vi. By substitution,

\[
\frac{1}{4}(2x - 1) = 0.5x - 0.25 \\
\frac{1}{2}x - \frac{1}{4} = 0.5x - 0.25 \\
\frac{1}{2}x - 0.5x = \frac{1}{4} - 0.25 \\
0 = 0
\]

vii. Multiply the first equation by 1.2 and the second equation by -4, and then use elimination.

\[
\begin{align*}
4.8x + 7.2y &= 10.8 \\
-4.8x - 7.2y &= -10.8 \\
0 &= 0
\end{align*}
\]

g. The lines in a consistent and dependent linear system have the same slope and the same \(y\)-intercept, or the equations are multiples of each other.

15. Find the slope of each segment and then use either endpoint to write an equation in point-slope form, \(y = y_1 + b(x - x_1)\).

\[
\overline{CD}: y = -3 + \frac{2}{3}(x - 1) \quad \text{or} \quad y = -1 + \frac{2}{3}(x - 4)
\]

\[
\overline{AB}: y = 2 + \frac{2}{3}(x + 2) \quad \text{or} \quad y = 4 + \frac{2}{3}(x - 1)
\]

\[
\overline{AD}: y = 2 - \frac{5}{3}(x + 2) \quad \text{or} \quad y = -3 - \frac{5}{3}(x - 1)
\]

\[
\overline{BC}: y = 4 - \frac{5}{3}(x - 1) \quad \text{or} \quad y = -1 - \frac{5}{3}(x - 4)
\]

[\(-9.4, 9.4, 1, -6.2, 6.2, 1\)]

16. a. \(\log_b xy = \log_b x + \log_b y = a + b\)

b. \(\log_b x^3 = 3 \log_b x = 3a\)

c. \(\log_b \sqrt{x} = \frac{3}{2} \log_b x = 2 \log_b y - \log_b x = \frac{2b}{a}\)

d. \(\frac{1}{2}b\). Let \(\log_b x = z\), so \(p^z = x\). Given that \(\log_b y = b\), then \(p^b = y\). By substitution, \(p^{\frac{1}{2}b} = p^b\), so \(2z = b\) and \(z = \frac{1}{2}b\). Therefore, \(\log_b \sqrt{x} = \frac{1}{2}b\).

e. \(\log_b \sqrt{x} = \log_b x^{1/2} = \frac{1}{2} \log_b x = \frac{1}{2}a\)

f. \(\log_b m y = \log_b m x + \log_b y = \frac{\log_b x}{\log_b m} + \frac{\log_b y}{\log_b m} = a + b\)

g. \(\log_b m y = \log_b m x + \log_b y = \frac{\log_b x}{\log_b m} + \frac{\log_b y}{\log_b m} = a + b\)

17. \(x = 2, y = \frac{1}{2}, z = -3\). First, eliminate one variable entirely to get two equations with two unknowns.

Multiply the first equation by 2 and then add it to the second equation to eliminate \(y\).

\[
\begin{align*}
2x + 4y + 2z &= 0 \\
3x - 4y + 5z &= -11 \\
5x + 7z &= -11
\end{align*}
\]

Multiply the second equation by -2 and then add it to the third equation to eliminate \(y\).

\[
\begin{align*}
-6x + 8y - 10z &= 22 \\
-2x - 8y - 3z &= -1 \\
-8x - 13z &= 23
\end{align*}
\]

Now you have two equations in \(x\) and \(z\). Multiply the first equation by 8 and the second equation by 5, and then add to eliminate \(x\) and solve for \(z\).

\[
\begin{align*}
40x + 56z &= -88 \\
-40x - 65z &= 115
\end{align*}
\]

\[
-9z = 27
\]

\[
z = -3
\]

Substitute \(-3\) for \(z\) in either of the two equations in \(x\) and \(z\) and solve for \(x\).

\[
\begin{align*}
5x + 7(-3) &= -11 \\
5x - 21 &= -11 \\
5x &= 10 \\
x &= 2
\end{align*}
\]

Substitute 2 for \(x\) and \(-3\) for \(z\) in any of the original equations to solve for \(y\).

\[
\begin{align*}
(2) + 2y + (3) &= 0 \\
2y &= 1 \\
y &= \frac{1}{2}
\end{align*}
\]

The solution to the system of equations is \(x = 2, y = \frac{1}{2}, \text{and } z = -3\).

EXTENSIONS

A. Answers will vary.

B. Matrix multiplication is distributive over matrix addition.

For example, if \(A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}\),

\(B = \begin{bmatrix} b_1 & b_{11} \\ b_2 & b_{22} \end{bmatrix}\), and \(C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\),

then \(A(B + C) = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \cdot \begin{bmatrix} b_1 + c_{11} & b_{12} + c_{12} \\ b_{21} + c_{21} & b_{22} + c_{22} \end{bmatrix} = \begin{bmatrix} a_1(b_{11} + c_{11}) + a_2(b_{21} + c_{21}) \\ a_1(b_{12} + c_{12}) + a_2(b_{22} + c_{22}) \end{bmatrix}\)
and \([A][B] + [A][C] = \]
\[
\begin{bmatrix}
a_1b_{11} + a_2b_{21} & a_1b_{12} + a_2b_{22} \\
a_1c_{11} + a_2c_{21} & a_1c_{12} + a_2c_{22} \\
a_1b_{11} + a_2b_{21} + a_1c_{11} + a_2c_{21} & a_1b_{12} + a_2b_{22} + a_1c_{12} + a_2c_{22}
\end{bmatrix}
\]
These are the same matrices by the distributive property.

**LESSON 6.3**

**Exercises**

1. a. \[
\begin{align*}
2x + 5y &= 8 \\
4x - y &= 6
\end{align*}
\]
\[
\begin{align*}
x - y + 2x &= 3 \\
x + 2y - 3z &= 1 \\
2x + y - z &= 2
\end{align*}
\]

2. a. \[
\begin{bmatrix}
1 & 2 & -1 \\
2 & -1 & 3 \\
2 & 1 & 1
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
2 & 1 & -1 \\
2 & 0 & 1 \\
2 & -1 & 3
\end{bmatrix}
\]

3. a. \[
\begin{align*}
1 & -1 & 2 \\
-1(1) + 1 & -1(-1) + 2 & -1(2) + (-3) \\
2 & 1 & -1
\end{align*}
\]
\[
\begin{align*}
3 & -1(3) + 1 \\
2
\end{align*}
\]

b. \[
\begin{align*}
1 & -1 & 2 \\
1 & 2 & 1 \\
-2(1) + 2 & -2(-1) + 1 & -2(2) + (-1) \\
0 & 3 & -5
\end{align*}
\]
\[
\begin{align*}
2 & -3 & 1 \\
2 & -1 & -2 \\
1
\end{align*}
\]

4. a. \[
\begin{bmatrix}
2 & 5 & 8 \\
4 & -1 & 6
\end{bmatrix}
\]

b. \[
\begin{align*}
2 & 5 & 8 \\
2 & -2(2) + 4 & -2(5) + (-1) & -2(8) + 6
\end{align*}
\]

5. a. \[
\begin{align*}
\frac{R_1}{2} & \rightarrow R_i \\
\frac{R_1}{2} & \rightarrow R_j
\end{align*}
\]
Here is one possible sequence of row operations to obtain a solution matrix.

The solution to the system is \(x = -31\), \(y = 24\), and \(z = -4\).
reduced to row-echelon form. Here, the row operation used yields an entire row of 0's in row 3. This means that the first equation is a multiple of the third; therefore, not enough information was given to find a single solution, and there are infinitely many solutions.

A system with infinitely many solutions is a dependent system.

d. \[
\begin{bmatrix}
3 & -7 & 1 & 5 \\
1 & -2 & 5 & 1 \\
6 & -2 & 2 & 14
\end{bmatrix}
\]

Here is one possible sequence of row operations to obtain a solution matrix.

The solution to the system is \( x = -1, y = 1 \), and \( z = 0 \).

c. \[
\begin{bmatrix}
3 & -1 & 1 & 7 \\
1 & -2 & 5 & 1 \\
6 & -2 & 2 & 14
\end{bmatrix}
\]

The augmented matrix cannot be reduced to row-echelon form (dependent system). In the process of finding a possible sequence of row operations to obtain a solution matrix, you will see that not all augmented matrices can be
6. a. Let \( x \) represent the number of goats, and let \( y \) represent the number of chickens.

\[
\begin{align*}
\begin{bmatrix} x + y & = & 47 \\ 4x + 2y & = & 118 \end{bmatrix} \quad \rightarrow \begin{bmatrix} 1 & 1 & | & 47 \\ 4 & 2 & | & 118 \end{bmatrix}
\end{align*}
\]

He has 12 goats and 35 chickens.

b. Enter the augmented matrix into your calculator and use the reduced row-echelon command.

The solution to the system is \( x = \frac{3}{2}, y = \frac{1}{3}, \) and \( z = -\frac{1}{6}. \)

7. 38°, 62°, 80°. Let \( x \) represent the measure of the smallest angle, let \( y \) represent the measure of the midsize angle, and let \( z \) represent the measure of the largest angle. Write a system of three linear equations and then rewrite it as an augmented matrix.

\[
\begin{align*}
\begin{bmatrix} x + y + z & = & 180 \\ z & = & 2x + 4 \\ y - x & = & 24 \end{bmatrix} & \quad \rightarrow \begin{bmatrix} 1 & 1 & 1 & | & 180 \\ -2 & 0 & 1 & | & 4 \\ -1 & 1 & 0 & | & 24 \end{bmatrix}
\end{align*}
\]

Solve the system using the reduced row-echelon command.

The angle measures are 38°, 62°, and 80°.

8. a. Supply: \( \dot{y} = 158.96x - 11412.4 \); demand: \( \dot{y} = -54.36x + 7617.27. \) Use your calculator to find these median-median lines.

b. Approximately (89, 2768).

c. \( \begin{bmatrix} 54.36 & 1 & 7617.27 \\ 158.96 & -1 & 11412.4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 89.2 \\ 0 & 1 & 2768.0 \end{bmatrix} \)

The solution verifies the answer to 8b. Use row reduction to solve the augmented matrix.

\[
\begin{align*}
R_2 + R_1 & \rightarrow R_1 \\
\begin{bmatrix} 213.32 & 0 & 19029.67 \\ 158.96 & -1 & 11412.4 \end{bmatrix} - R_2 \\
\begin{bmatrix} 1 & 0 & 89.2 \\ 0 & 1 & 2768.0 \end{bmatrix}
\end{align*}
\]

The solution matrix verifies that \( x = 89.2 \) and \( y = 2768.0. \)

9. \( y = 2x^2 - 3x + 4. \) Write three equations with the variables \( a, b, \) and \( c \) by substituting the three pairs of coordinates.

\[
a(1)^2 + b(1) + c = 3, \quad a + b + c = 3
\]

\[
a(4)^2 + b(4) + c = 24, \quad 16a + 4b + c = 24
\]

\[
a(-2)^2 + b(-2) + c = 18, \quad 4a - 2b + c = 18
\]

Create a system of three equations and write it as an augmented matrix.

\[
\begin{align*}
\begin{bmatrix} a + b + c & = & 3 \\ 16a + 4b + c & = & 24 \\ 4a - 2b + c & = & 18 \end{bmatrix} & \quad \rightarrow \begin{bmatrix} 1 & 1 & 1 & | & 3 \\ 16 & 4 & 1 & | & 24 \\ 4 & -2 & 1 & | & 18 \end{bmatrix}
\end{align*}
\]
Use the calculator's reduced row-echelon command to solve.

\[
\begin{bmatrix}
1 & 1 & 1 & 3 \\
16 & 4 & 1 & 24 \\
4 & 1 & 1 & 181 \\
\end{bmatrix}
\]
\[
\text{rref(A)} = \begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & 4 \\
\end{bmatrix}
\]

The solution is \( a = 2, \) \( b = -3, \) and \( c = 4, \) so the equation is \( y = 2x^2 - 3x + 4. \)

10. 3 full-page ads, 7 half-page ads, and 12 business-card-size ads. Let \( x \) represent the number of full-page ads, let \( y \) represent the number of half-page ads, and let \( z \) represent the number of business-card-size ads. There were 22 ads sold, so \( x + y + z = 22. \) Full-size ads sell for $200, half-page ads sell for $125, business-card-size ads sell for $20, and the total income was $1715, so \( 200x + 125y + 20z = 1715. \) There were four times as many business-card-size ads as full-page ads, so \( 4x = z, \) or \( 4x - z = 0. \) Write this system of equations as an augmented matrix.

\[
\begin{align*}
x + y + z &= 22 \\
200x + 125y + 20z &= 1715 \\
4x - z &= 0
\end{align*}
\]
\[
\begin{bmatrix}
1 & 1 & 1 & 22 \\
200 & 125 & 20 & 1715 \\
4 & 0 & -1 & 0 \\
\end{bmatrix}
\]

Use the calculator's reduced row-echelon command to solve.

\[
\begin{bmatrix}
1 & 1 & 1 & 3 \\
200 & 125 & 20 & 1715 \\
4 & 1 & 1 & 121 \\
\end{bmatrix}
\]
\[
\text{rref(A)} = \begin{bmatrix}
1 & 0 & 0 & 2 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & 4 \\
\end{bmatrix}
\]

The solution is \( x = 3, \) \( y = 7, \) and \( z = 12, \) or 3 full-page ads, 7 half-page ads, and 12 business-card-size ads.

11a. First plan: \( 12,500 + 0.05(12)(3500) = 14,600 \) Second plan: \( 6800 + 0.15(12)(3500) = 13,100 \)

b. Let \( x \) represent the number of tickets sold, and let \( y \) represent the income in dollars; \( y = 12500 + 0.05(12)x = 12500 + 0.6x. \)

c. \( y = 6800 + 0.15(12)x = 6800 + 1.8x \)

d. More than 4750 tickets. Find the number of tickets sold at which the two plans generate the same amount of pay. By substitution,

\[
12500 + 0.6x = 6800 + 1.8x, \text{ so } 5700 = 1.2x, \text{ and } x = 4750. \text{ The second plan will be the better choice if they sell more than 4750 tickets.} \]

e. The company should choose the first plan if they expect to sell fewer than 4750 tickets and the second if they expect to sell more than 4750 tickets.

12. a. (4, 2). The intersection point for the two linear equations is (4, 2).

b. Using point-slope form, \( y_1 = 2 + 2(x - 4), \) or \( y_1 = 2(x - 3), \) and \( y_2 = 2 - 0.75(x - 4), \) or \( y_2 = -1 - 0.75(x - 8). \) The slope of line \( y_1 \) is 2 and the slope of line \( y_2 \) is \( -\frac{3}{4} \) or \(-0.75\). Line \( y_1 \) contains the point (3, 0), line \( y_2 \) contains the point (8, -1), and both lines contain the point (4, 2).

13. \( \overline{AB} \): \( y = 6 + \frac{2}{3}(x - 4) \) or \( y = 4 + \frac{2}{3}(x - 1) \)

\( \overline{BC} \): \( y = 4 - \frac{2}{3}(x - 7) \) or \( y = 6 - \frac{2}{3}(x - 4) \)

\( \overline{CD} \): \( y = 1 + 3(x - 6) \) or \( y = 4 + 3(x - 7) \)

\( \overline{DE} \): \( y = 1 \)

\( \overline{AE} \): \( y = 4 - 3(x - 1) \) or \( y = 1 - 3(x - 2) \)

\[
\begin{bmatrix}
-4.7, 14.1, 1, -3.1, 9.3, 1
\end{bmatrix}
\]

14. a. \( [M] = \begin{bmatrix}
2 & 6 & 6 \\
1 & 1 & 3 \\
\end{bmatrix} \). The vertices of the triangle are at (2, 1), (6, 1), and (6, 3).

b. i. \( \begin{bmatrix}
1 & 1 & 3 \\
2 & 6 & 6 \\
\end{bmatrix} \). \( \triangle ABC \) is reflected across the line \( y = x. \)

\[
\begin{bmatrix}
0 & 1 & 2 & 6 & 6 \\
1 & 0 & 1 & 1 & 3
\end{bmatrix} \begin{bmatrix}
2 & 6 & 6
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 3
\end{bmatrix}
\]
**Lesson 6.4**

**Exercises**

1. a. 
\[
\begin{bmatrix}
3 & 4 \\
2 & -5
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
11 \\
-8
\end{bmatrix}
\]

b. 
\[
\begin{bmatrix}
1 & 2 & 1 \\
3 & -4 & 5 \\
-2 & -8 & -3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
0 \\
-11 \\
1
\end{bmatrix}
\]

c. 
\[
\begin{bmatrix}
5.2 & 3.6 \\
-5.2 & 2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
7 \\
8.2
\end{bmatrix}
\]

d. 
\[
\begin{bmatrix}
1 & 2 & -2 \\
3 & 2 & 5
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
3 \\
2
\end{bmatrix}
\]

2. a. 
\[
\begin{bmatrix}
5 & 2 \\
7 & 3
\end{bmatrix}
\begin{bmatrix}
1 \\
5
\end{bmatrix} =
\begin{bmatrix}
15 \\
22
\end{bmatrix}
\]

c. Not possible; the dimensions of the matrices are 1 \times 2 and 3 \times 2, so the inside dimensions are not the same.

3. a. 
\[
\begin{bmatrix}
a & b \\
c & d
\end{bmatrix} =
\begin{bmatrix}
3 & -7 \\
-2 & 8
\end{bmatrix}
\]

c. Set corresponding entries equal.
\[
a + 5c = -7 \\
b + 5d = 33
\]
\[
6a + 2c = 14 \\
6b + 2d = -26
\]
Treat the equations as two systems of equations. Use either substitution or elimination to solve each system.

\[
\begin{align*}
\begin{cases}
a + 5c &= -7 \\
6a + 2c &= 14
\end{cases} & \quad \text{A system in two variables, } a \text{ and } c.
\end{align*}
\]

\[
2a + 10c = -14
\]

Multiply the first equation by 2.

\[-30a - 10c = -70
\]

Multiply the second equation by -5.

\[-28a = -84
\]

Add the equations to eliminate c.

\[a = 3
\]

Divide both sides by -28.

\[(3) + 5c = -7
\]

Substitute 3 for a in the first equation to find c.

\[5c = -10
\]

Subtract 3 from both sides.

\[c = -2
\]

Divide both sides by 2.

The solution is \(a = 3\) and \(c = -2\). Use a similar procedure to find b and d.

\[
\begin{align*}
\begin{cases}
b + 5d &= 33 \\
6b + 2d &= -26
\end{cases} & \quad \text{A system in two variables, } b \text{ and } d.
\end{align*}
\]

\[2b + 10d = 66
\]

Multiply the first equation by 2.

\[-30b - 10d = 130
\]

Multiply the second equation by -5.

\[-28b = 196
\]

Add the equations to eliminate d.

\[b = -7
\]

Divide both sides by -28.

\[(7) + 5d = 33
\]

Substitute -7 for b in the first equation to find d.

\[5d = 40
\]

Add 7 to both sides.

\[d = 8
\]

Divide both sides by 5.

The solution is \(b = -7\) and \(d = 8\).

The four variables are \(a = 3\), \(b = -7\), \(c = -2\), and \(d = 8\).

\[
b \begin{bmatrix}
a \\
c
\end{bmatrix} = \begin{bmatrix}
\frac{-1}{14} & \frac{5}{28} \\
\frac{3}{14} & \frac{-1}{28}
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 5 \\
6 & 2
\end{bmatrix} \begin{bmatrix}
a \\
c
\end{bmatrix} = \begin{bmatrix}
1a + 5c \\
6a + 2c
\end{bmatrix} = \begin{bmatrix}
1b + 5d \\
6b + 2d
\end{bmatrix}
\]

Set corresponding entries equal.

\[a + 5c = 1
\]

\[b + 5d = 0
\]

\[6a + 2c = 0
\]

\[6b + 2d = 1
\]

Treat the equations as two systems of equations. Use either substitution or elimination to solve each system.

\[
\begin{align*}
\begin{cases}
a + 5c &= 1 \\
6a + 2c &= 0
\end{cases} & \quad \text{A system in two variables, } a \text{ and } c.
\end{align*}
\]

\[a = 1 - 5c
\]

Solve the first equation for a.

\[6(1 - 5c) + 2c = 0
\]

Substitute \((1 - 5c)\) for a in the second equation.

\[6 - 30c + 2c = 0
\]

Distribute.

\[-28c = -6
\]

Add \(-30c\) and 2c, and subtract 6 from both sides.

\[c = \frac{3}{14}
\]

Divide both sides by -28.

\[a + 5 \left(\frac{3}{14}\right) = 1
\]

Substitute \(\frac{3}{14}\) for c in the first equation.

\[a + \frac{15}{14} = 1
\]

Multiply \(5 \left(\frac{3}{14}\right)\).

\[a = -\frac{1}{14}
\]

Subtract \(\frac{15}{14}\) from both sides.

The solution is \(a = -\frac{1}{14}\) and \(c = \frac{3}{14}\). Use a similar procedure to find b and d.

\[
\begin{align*}
\begin{cases}
b + 5d &= 0 \\
6b + 2d &= 1
\end{cases} & \quad \text{A system in two variables, } b \text{ and } d.
\end{align*}
\]

\[b = -5d
\]

Solve the first equation for b.

\[6(-5d) + 2d = 1
\]

Substitute \(-5d\) for b in the second equation.

\[-30d + 2d = 1
\]

Multiply \(6(-5d)\).

\[-28d = 1
\]

Add \(-30d\) and 2d.

\[d = -\frac{1}{28}
\]

Divide both sides by -28.

\[b + 5 \left(\frac{-1}{28}\right) = 0
\]

Substitute \(-\frac{1}{28}\) for d in the first equation.

\[b - \frac{5}{28} = 0
\]

Multiply \(5 \left(-\frac{1}{28}\right)\).

\[b = \frac{5}{28}
\]

Add \(\frac{5}{28}\) to both sides.

The solution is \(b = \frac{5}{28}\) and \(d = -\frac{1}{28}\).

The four variables are \(a = -\frac{1}{14}\), \(b = \frac{5}{28}\), \(c = \frac{3}{14}\), and \(d = -\frac{1}{28}\).
4. a. Yes, the matrices are inverses of each other because the product is the identity matrix.

\[
\begin{bmatrix}
5 & 2 \\
7 & 3 \\
\end{bmatrix}
\begin{bmatrix}
3 & -2 \\
-7 & 5 \\
\end{bmatrix} =
\begin{bmatrix}
5(3) + 2(-7) & 5(-2) + 2(5) \\
7(3) + 3(-7) & 7(-2) + 3(5) \\
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

b. Yes, the matrices are inverses of each other because the product is the identity matrix.
(See equation at bottom of page.)

5. a. \[
\begin{bmatrix}
4 & -3 \\
-5 & 4 \\
\end{bmatrix}
\]
Write a matrix equation in the form \([A][A]^{-1} = [I]\). Let \([A]^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\).

\[
\begin{bmatrix}
4 & 3 \\
5 & 4 \\
\end{bmatrix}
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix} =
\begin{bmatrix}
4a + 3c & 4b + 3d \\
5a + 4c & 5b + 4d \\
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

Set corresponding entries equal.

\[
4a + 3c = 1 \quad 4b + 3d = 0 \\
5a + 4c = 0 \quad 5b + 4d = 1
\]

Treat the equations as two systems of equations. Use substitution, elimination, or row reduction to solve each system.

\[
\begin{align*}
4a + 3c &= 1 \\
5a + 4c &= 0 \\
\end{align*}
\]
a = 4 and c = -5

\[
\begin{align*}
4b + 3c &= 0 \\
5b + 4d &= 1 \\
\end{align*}
\]
b = -3 and d = 4

The four variables are \(a = 4\), \(b = -3\), \(c = -5\), and \(d = 4\), so \([A]^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}\).

Use your calculator to find \([A]^{-1}\) and check your answer.

\[
\begin{bmatrix}
\frac{4}{5} & \frac{3}{5} \\
\frac{3}{5} & \frac{4}{5} \\
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
\frac{1}{2} & -1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}, \text{ or } \\
\begin{bmatrix}
-0.16 & 0.8 & 0.1 \\
0.5 & -1 & 0 \\
0 & 0 & 0.3 \\
\end{bmatrix}
\]
Write a matrix equation in the form \([A][A]^{-1} = [I]\). Let \([A]^{-1} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\).

\[
\begin{bmatrix}
6 & 4 & -2 \\
3 & 1 & -1 \\
0 & 0 & 3 \\
\end{bmatrix}
\begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i \\
\end{bmatrix} =
\begin{bmatrix}
6a + 4d - 2g & 6b + 4e - 2h & 6c + 4f - 2i \\
3a + 1d - 1g & 3b + 1e - 1h & 3c + 1f - 1i \\
0a + 0d + 3g & 0b + 0e + 3h & 0c + 0f + 3i \\
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

Set corresponding entries equal.

\[
\begin{align*}
6a + 4d - 2g &= 1 \\
6b + 4e - 2h &= 0 \\
6c + 4f - 2i &= 0 \\
3a + 1d - 1g &= 0 \\
3b + 1e - 1h &= 1 \\
3c + 1f - 1i &= 0 \\
0a + 0d + 3g &= 0 \\
0b + 0e + 3h &= 0 \\
0c + 0f + 3i &= 1
\end{align*}
\]
Treat the equations as three systems of three equations. Use substitution, elimination, or row reduction to solve each system.

\[
\begin{align*}
6a + 4d - 2g &= 1 \\
3a + 1d - 1g &= 0; \ a = \frac{-1}{6}, \ d = \frac{1}{2}, \text{ and } g = 0 \\
3g &= 0 \\
6b + 4e - 2h &= 0 \\
3b + 1e - 1h &= 1; \ b = \frac{2}{3}, \ e = -1, \text{ and } h = 0 \\
3h &= 0 \\
3c + 1f - 1i &= 0 \\
6c + 4f - 2i &= 0; \ c = \frac{1}{9}, \ f = 0, \text{ and } i = \frac{1}{3} \\
3i &= 1
\end{align*}
\]

The nine variables are \(a = \frac{-1}{6}, \ b = \frac{2}{3}, \ c = \frac{1}{9}, \ d = \frac{1}{2}, \ e = -1, \ f = 0, \ g = 0, \ h = 0, \) and \(i = \frac{1}{3}, \) so

\[
[A]^{-1} = \begin{bmatrix}
-\frac{1}{6} & 2 & 1 \\
\frac{1}{2} & -1 & 0 \\
0 & 0 & \frac{1}{3}
\end{bmatrix}
\]

Use your calculator to find \([A]^{-1}\) and check your answer.
6. For 6a–d, let matrix \([A]\) represent the coefficients of the variables, matrix \([X]\) represent the variables, and matrix \([B]\) represent the constant terms. To find the solution to each system, multiply \([A]^{-1}[B]\).

a. \[
\begin{bmatrix}
8 & 3 \\
6 & 5 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
41 \\
39 \\
\end{bmatrix}; x = 4, y = 3
\]

Use your calculator to find the inverse of \([A]\).

\[
[A]^{-1} = 
\begin{bmatrix}
\frac{5}{22} & -\frac{3}{22} \\
-\frac{3}{11} & \frac{4}{11} \\
\end{bmatrix}
\]

Left-multiply both sides of the equation by the inverse to find the solution to the system of equations.

\[
\begin{bmatrix}
8 & 3 \\
6 & 5 \\
\end{bmatrix}
\begin{bmatrix}
\frac{5}{22} & -\frac{3}{22} \\
-\frac{3}{11} & \frac{4}{11} \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
\frac{5}{22} & -\frac{3}{22} \\
-\frac{3}{11} & \frac{4}{11} \\
\end{bmatrix}
\begin{bmatrix}
41 \\
39 \\
\end{bmatrix}
\]

By the definitions of inverse and identity matrix, you are left with only matrix \([X]\) on the left side of the equation. Complete the matrix multiplication to find the solution.

\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
\frac{5}{22} & -\frac{3}{22} \\
-\frac{3}{11} & \frac{4}{11} \\
\end{bmatrix}
\begin{bmatrix}
41 \\
39 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
\frac{5}{22} & -\frac{3}{22} \\
-\frac{3}{11} & \frac{4}{11} \\
\end{bmatrix}
\begin{bmatrix}
41 \\
39 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
4 \\
3 \\
\end{bmatrix}
\]

The solution to the system is \(x = 4\) and \(y = 3\). Substitute values back into the original equations to check the solution.

\[
8x + 3y = 41 \\
6x + 5y = 39
\]

\[
8(4) + 3(3) \neq 41 \\
6(4) + 5(3) = 39
\]

\[
32 + 9 \neq 41 \\
34 + 15 = 39
\]

\[
41 = 41 \\
39 = 39
\]

The solution checks.

b. \[
\begin{bmatrix}
11 & -5 \\
9 & 2 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
-38 \\
-25 \\
\end{bmatrix}; x = -3, y = 1
\]

Use your calculator to find \([A]^{-1}\).

\[
[A]^{-1} = 
\begin{bmatrix}
\frac{2}{67} & \frac{5}{67} \\
\frac{-9}{67} & \frac{11}{67} \\
\end{bmatrix}
\]

To find the solution to the system, multiply \([A]^{-1}[B]\).

\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
\frac{2}{67} & \frac{5}{67} \\
\frac{-9}{67} & \frac{11}{67} \\
\end{bmatrix}
\begin{bmatrix}
-38 \\
-25 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
\end{bmatrix} = 
\begin{bmatrix}
-3 \\
1 \\
\end{bmatrix}
\]

The solution to the system is \(x = -3\) and \(y = 1\). Substitute values back into the original equations to check the solution.

\[
11x - 5y = -38 \\
9x + 2y = -25
\]

\[
11(-3) - 5(1) \neq -38 \\
9(-3) + 2(1) \neq -25
\]

\[
-33 - 5 \neq -38 \\
-27 + 2 \neq -25
\]

\[
-38 = -38 \\
-25 = -25
\]

The solution checks.

c. \[
\begin{bmatrix}
2 & 1 & -2 \\
6 & 2 & -4 \\
4 & -1 & 3 \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = 
\begin{bmatrix}
1 \\
3 \\
5 \\
\end{bmatrix}; x = 0.5, y = 6, z = 3
\]

Find \([A]^{-1}\) on your calculator.

\[
[A]^{-1} = 
\begin{bmatrix}
-1 & 0.5 & 0 \\
17 & -7 & 2 \\
7 & -3 & 1 \\
\end{bmatrix}
\]

To find the solution to the system, multiply \([A]^{-1}[B]\).

\[
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = 
\begin{bmatrix}
-1 & 0.5 & 0 \\
17 & -7 & 2 \\
7 & -3 & 1 \\
\end{bmatrix}
\begin{bmatrix}
1 \\
3 \\
5 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
z \\
\end{bmatrix} = 
\begin{bmatrix}
0.5 \\
6 \\
3 \\
\end{bmatrix}
\]

The solution to the system is \(x = 0.5, y = 6,\) and \(z = 3\). Substitute values back into the original equations to check the solution.

\[
2x + y - 2z = 1 \\
2(0.5) + 6 - 2(3) \neq 1 \\
1 + 6 - 6 \neq 1
\]

\[
1 = 1
\]

\[
6x + 2y - 4z = 3 \\
6(0.5) + 2(6) - 4(3) \neq 3 \\
3 + 12 - 12 \neq 3
\]

\[
3 = 3
\]
\[ 4x - y + 3z = 5 \]
\[ 4(0.5) - 6 + 3(3) \frac{1}{2} = 5 \]
\[ 2 - 6 + 9 \frac{1}{2} = 5 \]
\[ 5 = 5 \]

The solution checks.

d. \[
\begin{bmatrix}
4 & 1 & 2 & -3 \\
-3 & 3 & -1 & 4 \\
5 & 4 & 3 & -1 \\
-1 & 2 & 5 & 1
\end{bmatrix}
\begin{bmatrix}
w \\
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
-16 \\
20 \\
-10 \\
-4
\end{bmatrix}
\]

\( w = -1, \ x = 1, \ y = -2, \ z = 3 \)

Find \([A]^{-1}\) on your calculator.

\[ [A]^{-1} = \begin{bmatrix}
-43 & -23 & 31 & -3 \\
23 & 15 & -19 & 1 \\
-15 & -11 & 3 & 11 \\
-35 & -29 & -14 & -58
\end{bmatrix} \]

To find the solution to the system, multiply \([A]^{-1}[B]\).

\[ \begin{bmatrix}
w \\
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
-16 \\
20 \\
-10 \\
-4
\end{bmatrix}
\begin{bmatrix}
-43 & -23 & 31 & -3 \\
23 & 15 & -19 & 1 \\
-15 & -11 & 3 & 11 \\
-35 & -29 & -14 & -58
\end{bmatrix}
\]

\[ \begin{bmatrix}
w \\
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
1 \\
-2 \\
3
\end{bmatrix}
\]

The solution to the system is \( w = -1, \ x = 1, \ y = -2, \) and \( z = 3 \). Substitute values back into the original equations to check the solution.

\[ 4w + x + 2y - 3z = -16 \]
\[ 4(-1) + 1 + 2(-2) - 3(3) \frac{1}{2} = -16 \]
\[ -4 + 1 - 4 - 9 \frac{1}{2} = -16 \]
\[ -16 = -16 \]

\[ 5w + 4x + 3y - z = -10 \]
\[ 5(-1) + 4(1) + 3(-2) - 3 \frac{1}{2} = -10 \]
\[ -5 + 4 - 6 - 3 \frac{1}{2} = -10 \]
\[ -10 = -10 \]

\[ -3w + 3x - y + 4z = 20 \]
\[ -3(-1) + 3(1) - (-2) + 4(3) \frac{1}{2} = 20 \]
\[ 3 + 3 + 2 + 12 \frac{1}{2} = 20 \]
\[ 20 = 20 \]

\[ -w + 2x + 5y + z = -4 \]
\[ -(-1) + 2(1) + 5(-2) + 3 \frac{1}{2} = -4 \]
\[ 1 + 2 - 10 + 3 \frac{1}{2} = -4 \]
\[ -4 = -4 \]

The solution checks.

7. a. Jolly rides cost $0.50, Adventure rides cost $0.85, and Thrill rides cost $1.50. Let \( j \) represent the cost in dollars of each Jolly ride, let \( a \) represent the cost in dollars of each Adventure ride, and let \( t \) represent the cost in dollars of each Thrill ride. Use the numbers of tickets and the prices paid by each person to write a system of three equations in three variables.

\[
\begin{bmatrix}
7j + 3a + 9t = 19.55 \\
9j + 10a = 13 \\
8j + 7a + 10t = 24.95
\end{bmatrix}
\Rightarrow [A][X] = [B] \rightarrow
\]

\[
\begin{bmatrix}
7 & 3 & 9 \\
9 & 10 & 0 \\
8 & 7 & 10
\end{bmatrix}
\begin{bmatrix}
j \\
a \\
t
\end{bmatrix}
= 
\begin{bmatrix}
19.55 \\
13 \\
24.95
\end{bmatrix}
\]

Solve the system by multiplying \([A]^{-1}[B]\).

The solution is \( j = 0.5, \ a = 0.85, \) and \( t = 1.5 \). Therefore Jolly rides cost $0.50, Adventure rides cost $0.85, and Thrill rides cost $1.50.

b. \( 10(0.5 + 0.85 + 1.5) = 28.50 \)

c. Carey would have been better off buying a ticket book because without it, the cost was $24.95 + $5.00, or $29.95.
8. $2300 at 6% and $2700 at 7.5%. Let $x$ represent the amount in dollars invested at 6%, and let $y$ represent the amount in dollars invested at 7.5%. The total amount invested was $5000, so $x + y = 5000$. The total interest after one year was $340.50, so 0.06x + 0.075y = 340.50$. Write the system of two equations as a matrix equation.

\[
\begin{bmatrix}
1 & 1 \\
0.06 & 0.075
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
5000 \\
340.5
\end{bmatrix}
\]

Solve the system by multiplying $[A]^{-1}[B]$.

\[
\begin{bmatrix}
1 & 1 & 1 \\
0.06 & 0.075 & 0
\end{bmatrix}
\begin{bmatrix}
5000 \\
340.5
\end{bmatrix}
= 
\begin{bmatrix}
110 \\
50
\end{bmatrix}
\]

The solution is $x = 2300$, $y = 2700$. Therefore the family invested $2300 at 6% and $2700 at 7.5%.

9. $20^\circ$, $50^\circ$, $110^\circ$. Let $x$ represent the measure of the smallest angle, let $y$ represent the measure of the midsize angle, and let $z$ represent the measure of the largest angle. Write a system of three equations and then rewrite it as a matrix equation.

\[
\begin{align*}
x + y + z &= 180 \\
y &= 30 + x \\
z &= 2y + 10
\end{align*}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 0 \\
0 & -2 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
180 \\
30 \\
10
\end{bmatrix}
\]

Solve the system by multiplying $[A]^{-1}[B]$.

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 0 \\
0 & -2 & 1
\end{bmatrix}
\begin{bmatrix}
180 \\
30 \\
10
\end{bmatrix}
= 
\begin{bmatrix}
20 \\
50 \\
110
\end{bmatrix}
\]

The solution is $x = 20$, $y = 50$, and $z = 110$. Therefore the angle measures are $20^\circ$, $50^\circ$, and $110^\circ$.

10. The price of a citron is 8; the price of a fragrant wood apple is 5. Let $c$ represent the price of a citron, and let $w$ represent the price of a wood apple. Write a system of two equations and a matrix equation.

\[
\begin{align*}
9c + 7w &= 107 \\
7c + 9w &= 101
\end{align*}
\]

Solve the system by multiplying $[A]^{-1}[B]$. Find $[A]^{-1}$ on your calculator.

\[
\begin{bmatrix}
1 & 1 \\
0.06 & 0.075
\end{bmatrix}
\begin{bmatrix}
107 \\
101
\end{bmatrix}
= 
\begin{bmatrix}
9 \\
7
\end{bmatrix}
\]

\[
\begin{bmatrix}
c \\
w
\end{bmatrix}
= 
\begin{bmatrix}
8 \\
5
\end{bmatrix}
\]

The solution is $(8, 5)$. Therefore the price of a citron is 8 and the price of a fragrant wood apple is 5.

11. $x = 0.0016$, $y = 0.0126$, $z = 0.0110$. Write a system of three equations and a matrix equation.

\[
\begin{align*}
47x + 470y &= 6 \\
280z + 47y &= 9 \\
x + z &= y
\end{align*}
\]

\[
\begin{bmatrix}
47 & 470 & 0 \\
0 & 470 & 280 \\
1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
6 \\
9 \\
0
\end{bmatrix}
\]

Solve the system by multiplying $[A]^{-1}[B]$. Find $[A]^{-1}$ on your calculator.

\[
\begin{bmatrix}
47 & 470 & 0 \\
0 & 470 & 280 \\
1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
6 \\
9 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
0.0016 \\
0.0126 \\
0.0110
\end{bmatrix}
\]

The solution is $x = 0.0016$, $y = 0.0126$, and $z = 0.0110$. Therefore the current flowing through each resistor is 0.0016, 0.0126, and 0.0110 amps.

12. An error message means the system is either dependent or inconsistent. In this system, the lines are the same because the first equation is a multiple of the second equation, so the system is dependent. You can multiply the second equation by 1.6 to get the first equation.
Here are possible row operations on the augmented matrix for finding the inverse.

Augmented matrix
\[
\begin{bmatrix}
4 & 3 & 1 & 0 \\
5 & 4 & 0 & 1 \\
\end{bmatrix}
\]

\[-R_1 + R_2 \rightarrow R_1 \]
\[
\begin{bmatrix}
1 & 1 & -1 & 1 \\
5 & 4 & 0 & 1 \\
\end{bmatrix}
\]

\[-5R_1 + R_2 \rightarrow R_2 \]
\[
\begin{bmatrix}
1 & 1 & -1 & 1 \\
0 & -1 & 5 & -4 \\
\end{bmatrix}
\]

\[R_2 + R_1 \rightarrow R_1 \]
\[
\begin{bmatrix}
1 & 0 & 4 & -3 \\
0 & 1 & -5 & 4 \\
\end{bmatrix}
\]

\[R_2 \rightarrow R_2 \]
\[
\begin{bmatrix}
1 & 0 & 4 & -3 \\
0 & 1 & -5 & 4 \\
\end{bmatrix}
\]

Therefore, \[
\begin{bmatrix}
4 & -3 \\
5 & 4 \\
\end{bmatrix}
\]
is the inverse.

b. \[
\begin{bmatrix}
-0.5 & 1.4 & 0.1 \\
0.5 & -1 & 0 \\
-1.5 & 2.3 & 0.3 \\
\end{bmatrix}
\], or \[
\begin{bmatrix}
\frac{5}{9} & \frac{13}{9} & \frac{1}{9} \\
\frac{1}{2} & -1 & 0 \\
-\frac{7}{6} & \frac{7}{3} & \frac{1}{3} \\
\end{bmatrix}
\]

Here are possible row operations on the augmented matrix for finding the inverse.

Augmented matrix
\[
\begin{bmatrix}
6 & 4 & -2 & 1 & 0 & 0 \\
3 & 1 & -1 & 0 & 1 & 0 \\
0 & 7 & 3 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[\frac{R_1}{2} \rightarrow R_1 \]
\[
\begin{bmatrix}
3 & 2 & -1 & \frac{1}{2} & 0 & 0 \\
3 & 1 & -1 & 0 & 1 & 0 \\
0 & 7 & 3 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[-R_2 + R_1 \rightarrow R_1 \]
\[
\begin{bmatrix}
0 & 1 & 0 & \frac{1}{2} & -1 & 0 \\
3 & 1 & -1 & 0 & 1 & 0 \\
0 & 7 & 3 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[-7R_1 + R_3 \rightarrow R_3 \]
\[
\begin{bmatrix}
0 & 1 & 0 & \frac{1}{2} & -1 & 0 \\
3 & 1 & -1 & 0 & 1 & 0 \\
0 & 0 & 3 & -\frac{7}{2} & 7 & 1 \\
\end{bmatrix}
\]

\[\frac{R_3}{3} \rightarrow R_3 \]
\[
\begin{bmatrix}
0 & 1 & 0 & \frac{1}{2} & -1 & 0 \\
3 & 1 & -1 & 0 & 1 & 0 \\
0 & 0 & 1 & -\frac{7}{6} & \frac{7}{3} & \frac{1}{3} \\
\end{bmatrix}
\]

Therefore, \[
\begin{bmatrix}
-0.5 & 1.4 & 0.1 \\
0.5 & -1 & 0 \\
-1.5 & 2.3 & 0.3 \\
\end{bmatrix}
\], is the inverse.

14. \[
[X] = \begin{bmatrix}
202.9 \\
228.6 \\
165.7 \\
\end{bmatrix}
\]

First, factor \([X]\) from the right side of the equation \([X] - [A][X] = [D]\) to get \((I - [A])[X] = [D]\).

Solve the equation \((I - [A])[X] = [D]\) by left-multiplying by the inverse of \((I - [A])\), or \([X] = (I - [A])^{-1}[D]\).

\[
[I] - [A] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix} - \begin{bmatrix}
0.2 & 0.2 & 0.1 \\
0.2 & 0.4 & 0.1 \\
0.1 & 0.2 & 0.3 \\
\end{bmatrix}
\]

\[
[I] - [A]^{-1} = \begin{bmatrix}
0.8 & -0.2 & -0.1 \\
-0.2 & 0.6 & -0.1 \\
-0.1 & -0.8 & 0.7 \\
\end{bmatrix}
\]

\[
([I] - [A])^{-1} = \begin{bmatrix}
1.43 & 0.57 & 0.29 \\
0.54 & 1.96 & 0.36 \\
0.36 & 0.64 & 1.57 \\
\end{bmatrix}
\]

\[
[X] = \begin{bmatrix}
1.43 & 0.57 & 0.29 \\
0.54 & 1.96 & 0.36 \\
0.36 & 0.64 & 1.57 \\
\end{bmatrix} \begin{bmatrix}
100 \\
80 \\
50 \\
\end{bmatrix} = \begin{bmatrix}
202.9 \\
228.6 \\
165.7 \\
\end{bmatrix}
\]

To fulfill consumer demand, $202.9 million worth of agriculture products, $228.6 million worth of manufacturing, and $165.7 million worth of services should be produced.
15. In each case, you can multiply the original equation by any number to create a consistent and dependent system. Possible answers:
   a. $2y = 4x + 8$
   b. $3y = -x - 9$
   c. $4x + 10y = 20$
   d. $2x - 4y = -12$

16. In each case, you can choose a different constant term and leave all other terms the same to create an inconsistent system. Possible answers:
   a. $y = 2x + 6$
   b. $y = -\frac{1}{3}x + 2$
   c. $2x + 5y = 0$
   d. $x - 2y = 6$

17. a. $[A] = \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$
   
   b. The value of $a_{22}$ is 0 because there are zero roads connecting Murray to itself.
   
   c. The matrix has reflection symmetry across the main diagonal.
   
   d. 5; 10. The matrix sum is twice the number of roads. Each road is counted twice in the matrix because it can be traveled in either direction.
   
   e. For example, if the road between Davis and Terre is one-way toward Davis, $a_{34}$ changes from 1 to 0. The matrix is no longer symmetric.

18. Given that $u_5 = 28$ and $u_7 = 80$, the common difference is $\frac{u_7 - u_5}{7 - 3} = \frac{80 - 28}{4} = \frac{52}{4} = 13$. So the first term, $u_1$, is $u_5 - 2(13) = 28 - 26 = 2$.

IMPROVING YOUR REASONING SKILLS

Because the cards in the deck are evenly divided (red or black), about half of the 60 people answered each question. About 50% of phone numbers have even last digits, so, of the 30 respondents who answered Question 1, roughly 15 answered yes and 15 answered no. The remainder of the respondents were answering Question 2, so approximately 37 – 15, or 22, of their responses must have been yes and 8 no. Therefore, of those who answered Question 2, about $\frac{22}{30}$ or 73%, answered yes.

You can also solve this problem with a probability tree diagram.

There are 37 yes answers, so $P(\text{yes}) = \frac{37}{60}$.

The tree diagram shows $P(\text{yes}) = .25 + .5p$.

$\frac{37}{60} = .25 + .5p$

$p = .73$

EXTENSIONS

A. See the solution to Take Another Look activity 1 on page 148.

B. Research results will vary.

LESSON 6.5

EXERCISES

1. a. $y < \frac{10 - 2x}{-5}$, or $y < -2 + 0.4x$

   $2x - 5y > 10$

   $-5y > 10 - 2x$

   $y < \frac{10 - 2x}{-5}$, or $y < -2 + 0.4x$

   b. $y < \frac{6 - 2x}{-12}$, or $y < -\frac{1}{2} + \frac{1}{6}x$

   $4(2 - 3y) + 2x > 14$

   $8 - 12y + 2x > 14$

   $-12y > 6 - 2x$

   $y < \frac{6 - 2x}{-12}$, or $y < -\frac{1}{2} + \frac{1}{6}x$

   2. a. 

   ![Graph]
b. Rewrite the linear inequality as \( y > \frac{3}{2} - x \).

c. \( y < 2 - 0.5x \). The \( y \)-intercept is 2, the slope is 0.5, and the graph is shaded below the dotted boundary line.

b. \( y \geq 3 + 1.5x \). The \( y \)-intercept is 3, the slope is 1.5, and the graph is shaded above the solid boundary line.

c. \( y > 1 - 0.75x \). The \( y \)-intercept is 1, the slope is -0.75, and the graph is shaded above the dotted boundary line.

d. \( y \leq 1.5 + 0.5x \). The \( y \)-intercept is 1.5, the slope is 0.5, and the graph is shaded below the solid boundary line.

4. \( y \geq 2.4x + 2 \) and \( y \leq -x^2 - 2x + 6.4 \). The graph is shaded above the solid boundary line, \( y = 2.4x + 2 \), and below the solid boundary of the parabola, \( y = -x^2 - 2x + 6.4 \).

5. Vertices: (0, 2), (0, 5), (2.752, 3.596), and (3.529, 2.353). Find the vertices by finding the intersection point of each pair of equations using substitution, elimination, or matrices, or by graphing the lines on your calculator and tracing to the intersection points. You only need to find the intersection points that are vertices of the feasible region.

\begin{align*}
\text{Equations} & \quad \text{Intersection Points} \\
\ y = 0.1x + 2 \text{ and } x = 0 & \quad (0, 2) \\
\ y = -0.51x + 5 \text{ and } x = 0 & \quad (0, 5) \\
\ y = -0.51x + 5 \text{ and } y = -1.6x + 8 & \quad (2.752, 3.596) \\
\ y = -1.6x + 8 \text{ and } y = 0.1x + 2 & \quad (3.529, 2.354) \\
\end{align*}

6. Vertices: (1, 0), (1.875, 0), (3.307, 2.291), (0.209, 0.791). Find the vertices by finding the intersection point of each pair of equations using substitution, elimination, or matrices, or by graphing the lines on your calculator and tracing to the intersection points. You only need to find the intersection points that are vertices of the feasible region.

\begin{align*}
\text{Equations} & \quad \text{Intersection Points} \\
\ y = 1 - x \text{ and } y = 0 & \quad (1, 0) \\
\ y = 1.6x - 3 \text{ and } y = 0 & \quad (1.875, 0) \\
\ y = 1.6x - 3 \text{ and } y = -(x - 2)^2 + 4 & \quad (3.307, 2.291) \\
\ y = -(x - 2)^2 + 4 \text{ and } y = 1 - x & \quad (0.209, 0.791) \\
\end{align*}

7. Vertices: (0, 4), (3, 0), (1, 0), (0, 2). Find the vertices by finding the intersection point of each pair of equations using substitution, elimination, or matrices, or by graphing the lines on your calculator and tracing to the intersection points. You only need to find the intersection points that are vertices of the feasible region.

In slope-intercept form, the inequalities are \( y \leq \frac{12 - 4x}{3} \), \( y \leq \frac{8 - 1.6x}{2} \), and \( y \geq 2 - 2x \).

\begin{align*}
\text{Equations} & \quad \text{Intersection Points} \\
\ 4x + 3y = 12 \text{ and } x = 0 & \quad (0, 4) \\
\ 4x + 3y = 12 \text{ and } y = 0 & \quad (3, 0) \\
\ 2x + y = 2 \text{ and } y = 0 & \quad (1, 0) \\
\ 2x + y = 2 \text{ and } x = 0 & \quad (0, 2) \\
\end{align*}

8. Vertices: (1, 0), (2.562, 1.562), (1.658, 2.5), (−1.5, 2.5). Find the vertices by finding the intersection point of each pair of equations using substitution, elimination, or matrices, or by graphing the lines on your calculator and tracing to the intersection points. You only need to find the intersection points that are vertices of the feasible region.

\begin{align*}
\text{Equations} & \quad \text{Intersection Points} \\
\ y = |x - 1| \text{ and } y = 0 & \quad (1, 0) \\
\ y = |x - 1| \text{ and } y = \sqrt{9 - x^2} & \quad (2.562, 1.562) \\
\ y = \sqrt{9 - x^2} \text{ and } y = 2.5 & \quad (1.658, 2.5) \\
\ y = |x - 1| \text{ and } y = 2.5 & \quad (-1.5, 2.5) \\
\end{align*}
9. Let $x$ represent the length in inches, and let $y$ represent the width in inches.

\[
\begin{align*}
xy & \geq 200 \\
xy & \leq 300 \\
x + y & \geq 33 \\
x + y & \leq 40
\end{align*}
\]

b. 

![Graph of linear equations]

c. i. No. Neither $(12.4, 16.3)$ nor $(16.3, 12.4)$ is in the feasible region.

ii. Yes. $(16, 17.5)$ and $(17.5, 16)$ are in the feasible region.

iii. No. Neither $(14.3, 17.5)$ nor $(17.5, 14.3)$ is in the feasible region.

10. a. $x > 27767$ km. Substitute 2 for $W$ and solve for $x$.

\[
2 = 57 \cdot \frac{6400^2}{(6400 + x)^2}
\]

\[
2(6400 + x)^2 = 57 \cdot 6400^2
\]

\[
(6400 + x)^2 = \frac{57 \cdot 6400^2}{2}
\]

\[
6400^2 + 2(6400)x + x^2 = 1167360000
\]

\[
40960000 + 12800x + x^2 = 1167360000
\]

$x^2 + 12800x - 1126400000 = 0$

Use the quadratic formula where $a = 1$, 
$b = 12800$, and $c = -1126400000$.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = 27767 \text{ or } x = -40567
\]

The altitude cannot be negative, so the astronaut will weigh less than 2 kg at altitudes greater than 27,767 km.

b. 50.5 kg. Substitute 400 for $x$ and solve for $W$.

\[
W = 57 \cdot \frac{6400^2}{(6400 + 400)^2} = 57(0.8858) = 50.5 \text{ kg}
\]

c. In theory no, because as the denominator grows larger, the value of the fraction approaches zero but never gets to zero.

11. a. $5x + 2y > 100$

b. $y < 10$

c. $x + y \leq 40$

d. Commonsense constraints: $x \geq 0$, $y \geq 0$

e. $(20, 0), (40, 0), (30, 10), (16, 10)$

12. a. $a = 3$, $b = 16$, $c = -12$. First, substitute the coordinate pairs in the quadratic equation $y = ax^2 + bx + c$ to create a system of three equations in three unknowns, $a$, $b$, and $c$. Set up a matrix equation in the form $[A][X] = [B]$.

\[
\begin{bmatrix}
-32 \\
7 \\
63
\end{bmatrix} = \begin{bmatrix}
4 & -2 & 1 \\
1 & 1 & 1 \\
9 & 3 & 1
\end{bmatrix} \begin{bmatrix}
a \\
b \\
c
\end{bmatrix}
\]

To find the solution to the system, multiply $[A]^{-1}[B]$.

\[
\begin{bmatrix}
4 & -2 & 1 \\
1 & 1 & 1 \\
9 & 3 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
-32 \\
7 \\
63
\end{bmatrix}
\]

Therefore, $a = 3$, $b = 16$, and $c = -12$.

b. $y = 3x^2 + 16x - 12$

c. Sample answer: Substitute each point into the equation to verify that it lies on the parabola. Or use a calculator table to make sure all the points fit the equation.
13. \( a = 100, b = 0.7 \). Substitute the given values for \( x \) and \( y \) in the exponential equation, \( y = ab^x \).

There will be two equations with two variables, \( a \) and \( b \). Solve the first equation for \( a \) in terms of \( b \), and use substitution to find the values of \( a \) and \( b \).

\[
34.3 = ab^5, \quad \text{so} \quad a = \frac{34.3}{b^5}
\]
\[
8.2 = ab^7
\]
\[
8.2 = \left( \frac{34.3}{b^5} \right) b^7
\]
\[
8.2 = 34.3 b^4
\]
\[
b^4 = 8.2 / 34.3
\]
\[
b \approx 0.7
\]

Substitute 0.7 for \( b \) in either equation to solve for \( a \).

\[
a = \frac{34.3}{(0.7)^5} = 100
\]

Therefore, \( a = 100 \) and \( b \approx 0.7 \).

14. \[
\left[ \begin{array}{ccc}
3 & -1 & 5 \\
-4 & 2 & 1
\end{array} \right] \rightarrow R_1 \rightarrow R_2
\]
\[
\left[ \begin{array}{ccc}
0 & 2 & \frac{23}{3} \\
-4 & 2 & 1
\end{array} \right]
\]
\[
\left[ \begin{array}{ccc}
1 & 0 & \frac{11}{2} \\
0 & 1 & \frac{23}{2}
\end{array} \right]
\]

15. a. 2 or 3 spores. Substitute 0 for \( x \):
\[
y = f(0) = (2.68)(3.84)^0 = 2.68.
\]

b. About 1,868,302 spores. Substitute 10 for \( x \):
\[
y = f(10) = (2.68)(3.84)^{10} = 1868301.824 = 1868302.
\]

c. \( x = \frac{\log_{2.68} y}{\log 3.84} \). Solve for \( x \) in terms of \( y \) to find the inverse function.

\[
y = (2.68)(3.84)^x
\]
\[
\frac{y}{2.68} = (3.84)^x
\]
\[
\log_{2.68} \frac{y}{2.68} = \log(3.84)^x
\]
\[
\log_{2.68} \frac{y}{2.68} = x \log(3.84)
\]
\[
x = \frac{\log y}{\log 3.84}
\]

d. After 14 hr 40 min. Substitute \( 1.0 \times 10^9 \) for \( y \).

\[
\log \frac{1.0 \times 10^9}{\log 3.84} \approx 14.67
\]

0.67, or \( \frac{2}{3} \), of an hour is 40 minutes, so the number of spores will exceed 1 billion after 14 hours 40 minutes.

**Improving Your Reasoning Skills**

The message says, "If eyes have no tears the soul can have no rainbow."

This coded message uses the same coding matrix \([E] \), so to decode, use the inverse, \([E]^{-1} = \left[ \begin{array}{cc} 2 & 1 \\ 1 & 1 \end{array} \right] \).

Begin by breaking the letters into groups of four letters each.

**CCFS** **LGGT** **QNYP** **OPII** **YUCB** **DKIC** **BYFB** **Eqww** **WURQ** **LPRE**

Convert the letters to numbers and multiply by the inverse of the coding matrix, \([E]^{-1} \), and then convert back into the corresponding letter in the alphabet.

**CCFS:** \([3 \ 6 \ 19] \rightarrow [9 \ 31 \ 25] \rightarrow [E]^{-1} \)

**LGGT:** \([12 \ 7 \ 20] \rightarrow [31 \ 34 \ 27] \rightarrow [E]^{-1} \)

**QNYP:** \([17 \ 25 \ 16] \rightarrow [48 \ 66 \ 41] \rightarrow [E]^{-1} \)

**OPII:** \([15 \ 9 \ 16] \rightarrow [46 \ 27 \ 18] \rightarrow [E]^{-1} \)

**YUCB:** \([25 \ 3 \ 2] \rightarrow [71 \ 8 \ 5] \rightarrow [E]^{-1} \)

**DKIC:** \([4 \ 9 \ 11] \rightarrow [19 \ 21 \ 12] \rightarrow [E]^{-1} \)

**S** **O**

**U** **L**
BYFB: \[
\begin{bmatrix}
2 & 25 \\
6 & 2
\end{bmatrix} = \begin{bmatrix}
29 & 27 \\
14 & 8
\end{bmatrix} \rightarrow
\]
\[
\begin{bmatrix}
3 & 1 \\
14 & 8
\end{bmatrix} = \begin{bmatrix}
C & A \\
N & H
\end{bmatrix}
\]

EQQW: \[
\begin{bmatrix}
5 & 17 \\
17 & 23
\end{bmatrix} = \begin{bmatrix}
27 & 22 \\
57 & 40
\end{bmatrix} \rightarrow
\]
\[
\begin{bmatrix}
1 & 22 \\
5 & 14
\end{bmatrix} = \begin{bmatrix}
A & V \\
E & N
\end{bmatrix}
\]

WURQ: \[
\begin{bmatrix}
23 & 21 \\
18 & 17
\end{bmatrix} = \begin{bmatrix}
67 & 44 \\
53 & 35
\end{bmatrix} \rightarrow
\]
\[
\begin{bmatrix}
15 & 18 \\
1 & 9
\end{bmatrix} = \begin{bmatrix}
O & R \\
A & I
\end{bmatrix}
\]

LPRE: \[
\begin{bmatrix}
12 & 16 \\
18 & 5
\end{bmatrix} = \begin{bmatrix}
40 & 28 \\
41 & 23
\end{bmatrix} \rightarrow
\]
\[
\begin{bmatrix}
14 & 2 \\
15 & 23
\end{bmatrix} = \begin{bmatrix}
N & B \\
O & W
\end{bmatrix}
\]

Arrange the letters in order and decipher the encoded message.

IFYE ESHE VENO TEAR STHE SOUL CANH AVEN ORAI NBOW

**LESSON 6.6**

**EXERCISES**

1. 

2. In 2a–d, substitute the coordinates of the vertices of the feasible region for x and y in each expression to find the maximum or minimum value.

a. \( \left( \frac{20}{3}, \frac{10}{3} \right) \)

\( (0, 10): 5(0) + 2(10) = 20 \)

\( \left( \frac{10}{3}, \frac{5}{3} \right): 5 \left( \frac{10}{3} \right) + 2 \left( \frac{5}{3} \right) = \frac{60}{3} = 20 \)

\( \left( \frac{20}{3}, \frac{10}{3} \right): 5 \left( \frac{20}{3} \right) + 2 \left( \frac{10}{3} \right) = \frac{120}{3} = 40 \)

The vertex \( \left( \frac{20}{3}, \frac{10}{3} \right) \) gives the largest value and thus maximizes the expression.

b. \( \left( \frac{10}{3}, \frac{5}{3} \right) \)

\( (0, 10): 0 + 3(10) = 30 \)

\( \left( \frac{10}{3}, \frac{5}{3} \right): \frac{10}{3} + 3 \left( \frac{5}{3} \right) = \frac{25}{3} \)

\( \left( \frac{20}{3}, \frac{10}{3} \right): \frac{20}{3} + 3 \left( \frac{10}{3} \right) = \frac{50}{3} \)

The vertex \( \left( \frac{10}{3}, \frac{5}{3} \right) \) gives the smallest value and thus minimizes the expression.

c. (0, 10)

\( (0, 10): 0 + 4(10) = 40 \)

\( \left( \frac{10}{3}, \frac{5}{3} \right): \frac{10}{3} + 4 \left( \frac{5}{3} \right) = \frac{30}{3} = 10 \)

\( \left( \frac{20}{3}, \frac{10}{3} \right): \frac{20}{3} + 4 \left( \frac{10}{3} \right) = \frac{60}{3} = 20 \)

The vertex (0, 10) gives the largest value and thus maximizes the expression.

d. (0, 10)

\( (0, 10): 5(0) + 10 = 10 \)

\( \left( \frac{10}{3}, \frac{5}{3} \right): \frac{5}{3} \left( \frac{10}{3} \right) + \frac{5}{3} = \frac{55}{3} \)

\( \left( \frac{20}{3}, \frac{10}{3} \right): \frac{5}{3} \left( \frac{20}{3} \right) + \frac{10}{3} = \frac{110}{3} \)

The vertex (0, 10) gives the smallest value and thus minimizes the expression.

e. It is not always obvious which point provides a maximum or minimum value.

3. 

Vertices: (5500, 5000), (3500, 16500), (10000, 30000), (35000, 5000). The vertex (10000, 30000) maximizes the function at \( P = 3800 \). Substitute the coordinates of each vertex into the function \( P = 0.08x + 0.10y \).

\( (5500, 5000): P = 0.08(5500) + 0.10(5000) = 940 \)

\( (5500, 16500): P = 0.08(5500) + 0.10(16500) = 2090 \)

\( (10000, 30000): P = 0.08(10000) + 0.10(30000) = 3800 \)

\( (35000, 5000): P = 0.08(35000) + 0.10(5000) = 3300 \)

The integer coordinates of the vertex (10000, 30000) maximize the function.
4. a. There are zero or more pairs of each species in the region.

b. The area required by species X plus the area required by species Y is no more than 180,000 m².

c. The total food requirement of species X plus the total food requirement of species Y is no more than 72,000 kg.

d. 

![Graph with points (0, 1034.5) and (1263.2, 315.8)]

The maximum number of nesting pairs is 1578. Evaluate the function \( N = x + y \) at each of the vertices.

\[(0, 1034.5): N = 0 + 1034.5 = 1034.5\]
\[(1263.2, 315.8): N = 1263.2 + 315.8 = 1579\]
\[(1500, 0): N = 1500 + 0 = 1500\]

The vertex \((1263.2, 315.8)\) gives the largest value, but you need integer points because \(x\) and \(y\) represent the number of nesting pairs of birds. Any of the integer points around \((1263.2, 315.8)\), which are \((1261, 317)\), \((1262, 316)\), \((1263, 315)\), \((1264, 314)\), and \((1265, 313)\), give the maximum number of nesting pairs, 1578.

5. a. Possible answer:

\[
\begin{align*}
  y &\geq 7 \\
  y &\leq \frac{7}{5}(x-3) + 6 \\
  y &\leq -\frac{7}{12}x + 13
\end{align*}
\]

![Graph with inequalities]

6. 12 sled dogs and 12 poodles for a maximum profit of $3360. Let \(x\) represent the number of sled dogs, and let \(y\) represent the number of poodles. Organize the constraint information into a table, and then write inequalities that reflect the constraints. Be sure to include commonsense constraints.

<table>
<thead>
<tr>
<th></th>
<th>Number of sled dogs, (x)</th>
<th>Number of poodles, (y)</th>
<th>Limiting value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum poodles</td>
<td></td>
<td>(y)</td>
<td>(\leq 20)</td>
</tr>
<tr>
<td>Maximum sled dogs</td>
<td>(x)</td>
<td></td>
<td>(\leq 15)</td>
</tr>
<tr>
<td>Food</td>
<td>6</td>
<td>2</td>
<td>(\leq 100)</td>
</tr>
<tr>
<td>Training</td>
<td>250</td>
<td>1000</td>
<td>(\leq 15000)</td>
</tr>
<tr>
<td>Profit</td>
<td>80</td>
<td>200</td>
<td>Maximize</td>
</tr>
</tbody>
</table>
The vertices of the feasible region are (0, 0), (15, 0), (15, 5), (12.7273, 11.8182), and (0, 15). Evaluate the function profit = 800x + 2000y at each of the vertices.

(0, 0): profit = 80(0) + 200(0) = 0
(15, 0): profit = 80(15) + 200(0) = 1200
(15, 5): profit = 80(15) + 200(5) = 1120
(12.7273, 11.8182): profit = 80(12.7273) + 200(11.8182) = 3381.82
(0, 15): profit = 80(0) + 200(15) = 3000

The vertex (12.7273, 11.8182) maximizes the function but does not represent whole values for dogs. The closest integer point to (12.7273, 11.8182) in the feasible region, which has the largest profit, is (12, 12): profit = 80(12) + 200(12) = 3360. Therefore, to maximize profits, they should raise 12 sled dogs and 12 poodles.

7. 5 radio minutes and 10 newspaper ads to reach a maximum of 155,000 people. This requires the assumption that people who listen to the radio are independent of people who read the newspaper, which is probably not realistic, and that the shop must buy both radio minutes and newspaper ads.

Let x represent the number of newspaper ads and y represent the number of minutes of radio advertising. Use the constraints to write this system of inequalities:

\[
\begin{align*}
x &\geq 4 \\
y &\geq 5 \\
50x + 100y &\leq 1000 \\
\text{people} &\leq 8000x + 15000y
\end{align*}
\]

The vertices of the feasible region are (0, 0) and (15, 15, 0). Test which point gives the largest number of people reached. Evaluate the function profit = 800x + 15000y at each of the vertices.

(4, 5): profit = 800(4) + 15000(5) = 107,000
(4, 8): profit = 800(4) + 15000(8) = 152,000
(10, 5): profit = 800(10) + 15000(5) = 155,000

Therefore, place 10 radio ads and 15 newspaper ads to reach a maximum of 155,000 people.

If the store places 20 radio ads and 5 newspaper ads, 160,000 people are reached. If it places 0 radio ads and 10 newspaper ads, 150,000 people are reached. Therefore the store could place 20 radio ads and no newspaper ads to reach the most people.

8. 600 barrels each of low-sulfur oil and high-sulfur oil for a minimum total cost of $19,920. Let x represent the number of barrels of low-sulfur oil, and let y represent the number of barrels of high-sulfur oil. Use the constraints to create this system of inequalities:

\[
\begin{align*}
x + y &\geq 1200 \\
0.02x + 0.06y &\leq 0.04(x + y) \\
x &\geq 0 \\
y &\geq 0
\end{align*}
\]

The vertices of the feasible region are (1200, 0) and (600, 600). Test which points minimize cost.

(1200, 0): cost = 18.50(1200) + 14.70(0) = 22,200
(600, 600): cost = 18.50(600) + 14.70(600) = 19,920

Therefore the minimum cost of $19,920 occurs when you use 600 barrels of low-sulfur oil and 600 barrels of high-sulfur oil.
9. 3000 acres of coffee and 4500 acres of cocoa for a maximum total income of $289,800. Let \( x \) represent the number of acres of coffee, and let \( y \) represent the number of acres of cocoa. Use the constraints to write this system:

\[
\begin{align*}
x + y &\leq 7500 \\
x &\geq 2450 \\
y &\geq 1230 \\
30x + 40y &\leq 270000
\end{align*}
\]

profit = 1.26(30x) + .98(40y) = 37.8x + 39.2y

[0, 8000, 1000, 0, 8000, 1000]

The vertices of the feasible region are (2450, 1230), (2450, 4912.5), (3000, 4500), and (6270, 1230). Test which point maximizes profit.

(2450, 1230): profit = 37.8(2450) + 39.2(1230) = 140826

(2450, 4912.5): profit = 37.8(2450) + 39.2(4912.5) = 92610 + 192570 = 285180

(3000, 4500): profit = 37.8(3000) + 39.2(4500) = 113400 + 176400 = 289800

(6270, 1230): profit = 37.8(6270) + 39.2(1230) = 237006 + 48216 = 285222

Therefore the maximum profit of $289,000 occurs when you plant 3000 acres of coffee and 4500 acres of cocoa.

10. a. Let \( x \) represent the length in inches, and let \( y \) represent the girth in inches.

\[
\begin{align*}
x + y &\leq 130 \\
x &\leq 108 \\
x &> 0 \\
y &> 0
\end{align*}
\]

b.

\[
\begin{array}{c}
0 \\
50 \\
100 \\
0 \\
50 \\
100
\end{array}
\]

11. a. \( x = -\frac{7}{11}, y = \frac{169}{11} \)

First, solve by using elimination.

\[
\begin{align*}
16x + 6y &= 82 \\
-27x - 6y &= -75
\end{align*}
\]

Multiply the first equation by 2.

Multiply the second equation by 3.

\[
\begin{align*}
-11x &= 7 \\
27x &= -11
\end{align*}
\]

Add the two equations.

\[
\begin{align*}
x &= -\frac{7}{11} \\
y &= \frac{169}{11}
\end{align*}
\]

Divide both sides by -11.

Substitute \(-\frac{7}{11}\) for \( x \) in the first equation and solve for \( y \).

\[
8 \left( -\frac{7}{11} \right) + 3y = 41
\]

\[
3y = 41 - \frac{56}{11} = \frac{451}{11} - \frac{56}{11} = \frac{507}{11}
\]

\[
y = \frac{169}{11}
\]

By elimination, \( x = -\frac{7}{11} \) and \( y = \frac{169}{11} \).

Second, solve the system using matrices. Enter the coefficients of the variables into matrix \([A]\), the variables into matrix \([X]\), and the constant terms into matrix \([B]\).

To find the solution to the system, find \([A]^{-1}[B]\).

\[
\begin{bmatrix}
8 & 3 \\
9 & 2
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
41 \\
25
\end{bmatrix}
\]

\[
[A]^{-1} =
\begin{bmatrix}
\frac{2}{11} & \frac{3}{11} \\
\frac{9}{11} & -\frac{8}{11}
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
\frac{2}{11} & \frac{3}{11} \\
\frac{9}{11} & -\frac{8}{11}
\end{bmatrix}
\begin{bmatrix}
41 \\
25
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
\frac{7}{11} \\
\frac{169}{11}
\end{bmatrix}
\]

By the matrix method, \( x = -\frac{7}{11} \) and \( y = \frac{169}{11} \).
b. \( x = -3.5, y = 74, z = 31 \). First, solve by using elimination.

\[
\begin{align*}
-4x - 2y + 4z &= -10 \\
6x + 2y - 4z &= 3 \\
2x &= -7 \\
x &= -3.5
\end{align*}
\]

Multiply the first equation by \(-2\).
The second equation.
Add the two equations.
Divide both sides by 2.

\[
\begin{align*}
2x + y - 2z &= 5 \\
4x - y + 3z &= 5 \\
6x + z &= 10
\end{align*}
\]
The first equation.
The third equation.
Add the first and third equations.

\[
6(-3.5) + z = 10
\]
Substitute \(-3.5\) for \(x\) in the resulting equation and solve for \(z\).

\[
z = 31
\]
Add 21 to both sides.

\[
2(-3.5) + y - 2(31) = 5
\]
Substitute \(-3.5\) for \(x\) and 31 for \(z\) in the first equation and solve for \(y\).

\[
y = 74
\]
Add 69 to both sides.

By elimination, \(x = -3.5, y = 74\), and \(z = 31\).

Second, solve the system by using matrices. Write the system in matrix form and multiply \([A]^{-1}[B]\) to find the solution.

\[
[A][X] = [B]
\]

\[
\begin{bmatrix}
2 & 1 & -2 \\
6 & 2 & -4 \\
4 & 1 & 3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
5 \\
3 \\
5
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
-1 & 0.5 & 0 \\
17 & -7 & 2 \\
7 & -3 & 1
\end{bmatrix}
\begin{bmatrix}
5 \\
3 \\
5
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= \begin{bmatrix}
-3.5 \\
74 \\
31
\end{bmatrix}
\]

By the matrix method, \(x = -3.5, y = 74\), and \(z = 31\).

12. 

\[
\begin{align*}
x &= 2 \\
y &\leq 5 \\
x + y &\geq 3 \\
2x - y &\leq 9
\end{align*}
\]

13. 

\[
\begin{pmatrix}
2 & -1 & \frac{1}{2} \\
5 & -2 & -5
\end{pmatrix}
\]

14. a. 

\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{bmatrix}
-6 \\
-12.5
\end{bmatrix}
\]

The solution checks.

15. \( y = 2x - \frac{1}{2} \) or \( y = -\frac{1}{4}x^2 - \frac{3}{2} \). The parabola, with parent function \( y = x^2 \), is reflected across the \(x\)-axis, shrunk vertically by a factor of \(\frac{1}{4}\), and translated down \(\frac{3}{2}\) units.

**Extensions**

A. See the solution to Take Another Look activity 3 on page 148.

B. Research results will vary.

**CHAPTER 6 REVIEW**

**Exercises**

1. a. The matrices are impossible to add because the dimensions are not the same.

b. 

\[
\begin{bmatrix}
-3 & -1 & 7 & -0 \\
6 & 5 & 4 & -2
\end{bmatrix}
= \begin{bmatrix}
-4 & 7 \\
1 & 2
\end{bmatrix}
\]

c. 

\[
\begin{bmatrix}
4(-3) & 4(1) & 4(2) \\
4(2) & 4(3) & 4(-2)
\end{bmatrix}
= \begin{bmatrix}
-12 & 4 & 8 \\
8 & 12 & -8
\end{bmatrix}
\]

d. 

\[
\begin{bmatrix}
1(-3) + 0(2) & 1(1) + 0(3) & 1(2) + 0(-2) \\
5(-3) + 2(2) & 5(1) + 2(3) & 5(2) + 2(-2)
\end{bmatrix}
= \begin{bmatrix}
-3 & 1 & 2 \\
-11 & 11 & 6
\end{bmatrix}
\]

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CHAPTER 5  141
e. The matrices are impossible to multiply because the inside dimensions do not match.

f. \[
\begin{bmatrix}
1(-3) + (-2)(2) & 1(1) + (-2)(3) & 1(2) + (-2)(-2)
\end{bmatrix} = \begin{bmatrix}
-7 & -5 & 6
\end{bmatrix}
\]

2. a. \[
\begin{bmatrix}
0.8 & -0.6 \\
0.2 & -0.4
\end{bmatrix}
\]

Write an equation in the form \([A][A]^{-1} = [I]\).

Let \([A]^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\).

\[
\begin{bmatrix} 2 & -3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a - 3c & 2b - 3d \\ 1a - 4c & 1b - 4d \end{bmatrix}
\]

\[
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

Set corresponding entries equal.

\[2a - 3c = 1 \quad 2b - 3d = 0 \]
\[a - 4c = 0 \quad b - 4d = 1\]

Treat the equation as two systems of equations. Use substitution or elimination to solve.

\[
\begin{bmatrix} 2a - 3c = 1 \\ a - 4c = 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2b - 3d = 0 \\ b - 4d = 1 \end{bmatrix}
\]

2. b. \[
\begin{bmatrix}
3 & 16 \\
85 & 85
\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}
18 \\
85
\end{bmatrix}
\]

\[
\begin{bmatrix}
29 \\
85
\end{bmatrix}
\]

\[
\begin{bmatrix}
-0.0353 \\
0.2118 \\
-0.3765
\end{bmatrix}
\]

Use your calculator to find \([A]^{-1}\).

3. a. \(x = 2.5, y = 7\). Write the system as an augmented matrix.

\[
\begin{bmatrix}
8x - 5y = -15 \\
6x + 4y = 43
\end{bmatrix} \Rightarrow \begin{bmatrix}
8 & -5 & -15 \\
6 & 4 & 43
\end{bmatrix}
\]

Here is one possible sequence of row operations to obtain a solution matrix.

\[
4R_1 + 5R_2 \rightarrow R_1 \quad \begin{bmatrix}
62 & 0 & 155 \\
6 & 4 & 43
\end{bmatrix}
\]

\[
R_1 \quad \begin{bmatrix}
1 & 0 & 2.5 \\
6 & 4 & 43
\end{bmatrix}
\]

\[-6R_1 + R_2 \rightarrow R_2 \quad \begin{bmatrix}
1 & 0 & 2.5 \\
0 & 4 & 28
\end{bmatrix}
\]

\[
R_2 \quad \begin{bmatrix}
1 & 0 & 2.5 \\
0 & 1 & 7
\end{bmatrix}
\]

The solution is \(x = 2.5\) and \(y = 7\).

b. \(x = 1.22, y = 6.9, z = 3.4\). Write the system as an augmented matrix.

\[
\begin{bmatrix}
5x + 3y - 7z = 3 \\
10x - 4y + 6z = 5 \\
15x + y - 8z = -2
\end{bmatrix} \Rightarrow \begin{bmatrix}
5 & 3 & -7 \\
10 & -4 & 6 \\
15 & 1 & -8
\end{bmatrix}
\]
Here is one possible sequence of row operations to obtain a solution matrix.

\[
\begin{align*}
-2R_1 + R_2 & \rightarrow R_2 \\
-3R_1 + R_3 & \rightarrow R_3 \\
R_2 \div 10 & \rightarrow R_2 \\
8R_2 + R_3 & \rightarrow R_3 \\
R_3 \div 3 & \rightarrow R_3 \\
-3R_2 + R_1 & \rightarrow R_1 \\
2R_3 + R_2 & \rightarrow R_3 \\
R_3 + R_1 & \rightarrow R_3 \\
R_1 \div 5 & \rightarrow R_1 \\
\end{align*}
\]

The solution is \( x = 1.22, y = 6.9, \) and \( z = 3.4. \)

4. In each problem, enter the coefficients of the variables into matrix \([A]\), the variables into matrix \([X]\), and the constant terms into matrix \([B]\). To find the solution to the system, multiply \([A]^{-1}\) by \([B]\).

a. \( x = 2.5, y = 7 \)
\[
[A][X] = [B]
\]
\[
\begin{bmatrix}
8 & -5 \\
6 & 4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
-15 \\
43
\end{bmatrix}
\]
\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
\frac{2}{31} & \frac{5}{62} \\
\frac{3}{31} & \frac{4}{31}
\end{bmatrix}
\begin{bmatrix}
-15 \\
43
\end{bmatrix}
\]
\[
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
2.5 \\
7
\end{bmatrix}
\]

The solution to the system is \( x = 2.5 \) and \( y = 7. \)

b. \( x = 1.22, y = 6.9, z = 3.4 \)
\[
[A][X] = [B]
\]
\[
\begin{bmatrix}
5 & 3 & 7 \\
10 & -4 & 6 \\
15 & 1 & -8
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
3 \\
5 \\
-2
\end{bmatrix}
\]
\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
13 & 17 & -1 \\
75 & 150 & 15 \\
-13 & 30 & 3
\end{bmatrix}
\begin{bmatrix}
3 \\
5 \\
-2
\end{bmatrix}
\]

The solution to the system is \( x = 1.22, y = 6.9, \) and \( z = 3.4. \)

5. a. Consistent and independent. By graphing, you can see that the system of equations has only one solution.

b. Consistent and dependent. Rewrite the first equation as \( y = \frac{1}{4}x + 3 \) by distributing \( \frac{1}{4} \) and combining like terms. You can see that it is the same as the second equation, \( y = -\frac{5}{3}x + 3. \) If you graph the two equations, there will be only one line.

c. Inconsistent. Rewrite the equations in intercept form: \( y = \frac{4}{3}x - \frac{2}{3} \) and \( y = \frac{13}{9}x - \frac{2}{3} \). The slopes of both lines are \(-\frac{2}{3}\) and the y-intercepts are different. By graphing, you can verify that the lines are parallel.

d. Inconsistent. If you rewrite the first equation using decimals or the second equation using fractions, you will see that the coefficients of the variables are the same but the constant term is different. Therefore the lines are parallel.
6. a.

Vertices: (0, 4), (2.625, 2.25), (3, 0), (0, 2)
Find the vertex that maximizes $1.65x + 5.2y$.

(0, 4): $1.65(0) + 5.2(4) = 20.8$
(2.625, 2.25): $1.65(2.625) + 5.2(2.25) = 16.03125$
(3, 0): $1.65(3) + 5.2(0) = 4.95$
(0, 2): $1.65(0) + 5.2(2) = 12.05$

The maximum occurs at (0, 4).

b.

Vertices: (0, 0), (0, 40), (30, 20), (38, 12), (44, 0)
Find the vertex that maximizes $6x + 7y$.

(0, 0): $6(0) + 7(0) = 0$
(0, 40): $6(0) + 7(40) = 280$
(30, 20): $6(30) + 7(20) = 320$
(38, 12): $6(38) + 7(12) = 312$
(44, 0): $6(44) + 7(0) = 264$

The maximum occurs at (30, 20).

7. About 4.4 yr, or about 4 yr 5 mo. Let $x$ represent the number of years, and let $y$ represent the cost of the water heater.

Cost for old unit: $y = 300 + 75x$
Cost for new unit: $y = 500 + 0.40(75x)$

Write the system as an augmented matrix.

\[
\begin{bmatrix}
-75 & 1 & 300 \\
-50 & 1 & 500
\end{bmatrix}
\]

Use your calculator to find the reduced row-echelon form.

\[
\begin{bmatrix}
1 & 0 & 4.4 \\
0 & 1 & 633.3
\end{bmatrix}
\]

The solution to the system indicates that after about 4.4 yr, both water heaters will cost the same, $633.33. Therefore a new heater will pay for itself in about 4.4 yr.

8. a. Let $x$ represent the number of parts of the first pre-mixed color, let $y$ represent the number of parts of the second pre-mixed color, and let $z$ represent the number of parts of the third pre-mixed color. The correct portion of red is given by the equation $2x + 1y + 3z = 5$.

b. $4x + 0y + 1z = 6$ for yellow; $0x + 2y + 1z = 2$ for black

c. $x = \frac{11}{8}$, $y = \frac{3}{4}$, $z = \frac{1}{2}$. Solve the system of equations using matrices.

\[
\begin{bmatrix}
2x + 1y + 3z = 5 \\
4x + 0y + 1z = 6 \\
0x + 2y + 1z = 2
\end{bmatrix} \rightarrow \begin{bmatrix}
2 & 1 & 3 & | & 5 \\
4 & 0 & 1 & | & 6 \\
0 & 2 & 1 & | & 2
\end{bmatrix}
\]

Use your calculator to find $[A]^{-1}[B]$.

\[
[A]^{-1} = \begin{bmatrix}
12 & 1 & 3 \\
4 & 2 & 1
\end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
15 \\
21
\end{bmatrix}
\]

\[
[A]^{-1}[B] = \begin{bmatrix}
11.375 \\
1.5
\end{bmatrix}
\]

The solution to the system is $x = \frac{11}{8}$, $y = \frac{3}{4}$, and $z = \frac{1}{2}$.

d. The least common denominator of the fractions $\frac{11}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$ is 8. Using 8 as a scalar multiplier, $8\left(\frac{11}{8}\right) = 11$, $8\left(\frac{3}{4}\right) = 6$, and $8\left(\frac{1}{2}\right) = 4$.

e. 11 parts of the first pre-mixed color, 6 parts of the second pre-mixed color, and 4 parts of the third pre-mixed color will yield the particular color needed.

\[
\begin{bmatrix}
.92 & .08 & 0 \\
.12 & .82 & .06 \\
0 & .15 & .85
\end{bmatrix}
\]

b. Set up a matrix equation using $[0.80 0.60 0.70]$ as the initial matrix to find the populations in the dorms after each month. Recursively multiply by the transition matrix to find the populations in the dorms in the given month.
i. October:

\[
\begin{bmatrix}
.92 & .08 & 0 \\
.12 & .82 & .06 \\
0 & .15 & .85
\end{bmatrix}
\]

Mozart: 80(.92) + 60(.12) = 80.8 \approx 81

Picasso: 80(.08) + 60(.82) + 70(.15) = 66.1 \approx 66

Hemingway: 60(.06) + 70(.85) = 63.1 \approx 63

ii. November:

\[
\begin{bmatrix}
.92 & .08 & 0 \\
.12 & .82 & .06 \\
0 & .15 & .85
\end{bmatrix}
\]

Mozart: 81(.92) + 66(.12) = 82.44 \approx 82

Picasso: 81(.08) + 66(.82) + 70(.15) = 70.05 \approx 70

Hemingway: 66(.06) + 63(.85) = 57.51 \approx 58

iii. May: Because May is eight months later, raise the transition matrix to the eighth power and multiply it by the initial matrix.

\[
\begin{bmatrix}
.92 & .08 & 0 \\
.12 & .82 & .06 \\
0 & .15 & .85
\end{bmatrix}^8 \approx
\begin{bmatrix}
94 & 76 & 40
\end{bmatrix}
\]

Therefore, in May, the dorm population is 94 students in Mozart, 76 students in Picasso, and 40 students in Hemingway.

10. They should make 4 shawls and 2 blankets to make the maximum profit of $104. Let \( x \) represent the number of shawls, and let \( y \) represent the number of blankets. Use the constraints to write this system:

\[
\begin{align*}
1x + 2y & \leq 8 \\
1x + 1y & \leq 6 \\
1x + 4y & \leq 14 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

Maximize: \( 16x + 20y \)

Vertices: (0, 0), (6, 0), (4, 2), (2, 3), (0, 3.5). Test which point maximizes profit.

(0, 0): \( 16(0) + 20(0) = 0 \)

(6, 0): \( 16(6) + 20(0) = 96 \)

(4, 2): \( 16(4) + 20(2) = 64 + 40 = 104 \)

(2, 3): \( 16(2) + 20(3) = 92 \)

(0, 3.5): \( 16(0) + 20(3.5) = 70 \)

The maximum occurs at (4, 2).

11. a. \( a < 0; p < 0; d > 0 \)

b. \( a > 0; p > 0; d \) cannot be determined.

c. \( a > 0; p = 0; d < 0 \)

12. a. \( f(-3) = 3 \)

b. \( -5 \) or approximately 3.5

c. It is a function because no vertical line crosses the graph in more than one place.

d. \( -6 \leq x \leq 5 \)

e. \( -2 \leq y \leq 4 \)

13. 20 students in second period, 18 students in third period, and 24 students in seventh period. Let \( x \) represent the number of students in second period, let \( y \) represent the number of students in third period, and let \( z \) represent the number of students in seventh period. Write a system of equations from the information given and solve it by any method.

For example, rewrite the system of three equations in matrix form and multiply \( [A]^{-1} [B] \) to find the solution to the system.

\[
\begin{align*}
\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z &= 22 \\
\frac{1}{4}x + \frac{1}{2}y + \frac{1}{6}z &= 18 \\
\frac{1}{4}x + \frac{1}{6}y + \frac{7}{12}z &= 22
\end{align*}
\]

\[
\begin{bmatrix}
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{2} & \frac{1}{6} \\
\frac{1}{4} & \frac{1}{6} & \frac{7}{12}
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
22 \\
18 \\
22
\end{bmatrix}
\]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\frac{1}{11}
\begin{bmatrix}
38 & -2 & -10 \\
-15 & 3 & -3 \\
-12 & 0 & 24
\end{bmatrix}
\begin{bmatrix}
22 \\
18 \\
22
\end{bmatrix}
\]
The solution is $x = 20$, $y = 18$, and $z = 24$. There are 20 students in second period, 18 students in third period, and 24 students in seventh period.

14. a. Let $x$ represent the year, and let $y$ represent the number of barrels per day in millions.

\[ [1950, 2010, 10, 0, 3, 1] \]

b. $M_1(1965, 0.89), M_2(1990, 1.55), M_3(1998.5, 1.945); \hat{y} = 0.0315x - 61.04$. To find $M_{1}, M_{2},$ and $M_{3}$, divide the data into three groups (2-3-3), and find the median of each group. Use your calculator to find the best approximation for the median-median line.

c. 2.023 million barrels per day. Using the median-median line, substitute 2002 for $x$: $\hat{y} = 0.0315(2002) - 61.04 = 2.023$.

15. a. $x = 245$

$log_{10} 35 + log_{10} 7 = log_{10} x$

$log_{10} (35 \cdot 7) = log_{10} x$

$35 \cdot 7 = x$

$x = 245$

b. $x = 20$

$log_{10} 500 - log_{10} 25 = log_{10} x$

$log_{10} \frac{500}{25} = log_{10} x$

$\frac{500}{25} = x$

$x = 20$

c. $x = \frac{1}{2}$

$log_{10} \sqrt{\frac{1}{8}} = x log_{10} 8$

$log_{10} \frac{1}{8} = log_{10} 8^x$

$\frac{1}{8} = 8^x$

$(8^{-1})^{1/2} = 8^x$

$8^{-1/2} = 8^x$

$x = \frac{-1}{2}$

d. $x = \frac{\log_{10} \frac{37000}{15}}{\log_{10} 9.4} \approx 3.4858$

$15(9.4)^x = 37000$

$(9.4)^x = \frac{37000}{15}$

$log_{10} (9.4)^x = log_{10} \left( \frac{37000}{15} \right)$

$x log_{10} (9.4) = log_{10} \left( \frac{37000}{15} \right)$

$x = \frac{log_{10} \frac{37000}{15}}{log_{10} 9.4} \approx 3.4858$

e. $x = 21$

$\sqrt[3]{(x + 6)} + 18.6 = 21.6$

$\sqrt[3]{x + 6} = 3$

$(x + 6)^{1/3} = 3$

$(x + 6)^{1/3} = 3$

$x + 6 = 27$

$x = 21$

f. $x = \frac{\log_{10} 342}{\log_{10} 36} \approx 1.6282$

$log_{10} 342 = 2x$

$6^x = 342$

$log_{10} 6^x = log_{10} 342$

$x log_{10} 6^x = log_{10} 342$

$x = \frac{log_{10} 342}{log_{10} 6^2}$

$x = \frac{log_{10} 342}{log_{10} 36} \approx 1.6282$

16. a. $100\% - 15\% = 85\%$
<table>
<thead>
<tr>
<th>Distance (ml)</th>
<th>Percentage of original signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>85</td>
</tr>
<tr>
<td>20</td>
<td>72.3</td>
</tr>
<tr>
<td>30</td>
<td>61.4</td>
</tr>
<tr>
<td>40</td>
<td>52.2</td>
</tr>
<tr>
<td>50</td>
<td>44.4</td>
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<td>37.7</td>
</tr>
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<td>70</td>
<td>32.1</td>
</tr>
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<td>80</td>
<td>27.29</td>
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<td>90</td>
<td>23.19</td>
</tr>
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<td>100</td>
<td>19.71</td>
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<td>16.76</td>
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<td>120</td>
<td>14.24</td>
</tr>
<tr>
<td>130</td>
<td>12.11</td>
</tr>
<tr>
<td>140</td>
<td>10.29</td>
</tr>
</tbody>
</table>

c. 280 ml. Use Home screen recursion to find that at a distance of 280 ml, 1.06% of the original signal strength is left.

17. a. \( y = 50(0.72)^{x-4} \) or \( y = 25.92(0.72)^{x-6} \). Using the point-ratio method, substitute the coordinates of the points into the point ratio form, \( y = y_1 \cdot b^{x-x_1} \).

\[ y = 50b^{x-4} \quad \text{and} \quad y = 25.92b^{x-6} \]

Use substitution to combine the two equations and solve for \( b \).

\[
50b^{x-4} = 25.92b^{x-6} \\
50 &= \frac{25.92}{b^{x-6}} \\
b^{x-4} &= \frac{25.92}{50} \\
b^{x-4} &= \frac{25.92}{50} \\
(b^{x-4})^{(x-4)} &= 0.5184 \\
b^2 &= 0.5184 \\
(b^2)^{1/2} &= (0.5184)^{1/2} \\
b &= 0.72
\]

Substitute 0.72 for \( b \) in either of the two original equations.

\( y = 50(0.72)^{x-4} \) or \( y = 25.92(0.72)^{x-6} \)

18. a. Sample answer using a bin width of 10:

b. Skewed left

c. Using the 1.5 \cdot IQR method, 54, 55, and 79 are outliers. Using the 2s method, only 54 and 55 are outliers.

\[ 1.5(Q_3 - Q_1) = 1.5(95 - 89) = 1.5(6) = 9, \quad \text{so} \quad Q_3 + 9 = 95 + 9 = 104, \quad \text{and} \quad Q_1 - 9 = 89 - 9 = 90. \]

There are no values greater than 104, and there are three values, 54, 55, and 79, below 90, which are therefore outliers.

The standard deviation is 13. Adding 2s, or 26, to the mean, 87.4, gives 113.4, and subtracting 26 from the mean gives 61.4. Therefore only 54 and 55 are outliers below 61.4.

d. 67th percentile. There are 12 weights that are lower than 94 kg when the data are put in increasing order, so \( \frac{12}{18} \approx .67, \) or 67%.

19. a. A translation right 5 units and down 2 units

b. A reflection across the x-axis and a vertical stretch by a factor of 2
c. \(-1 \cdot [P] = \begin{bmatrix} 2 & 1 & 0 & -1 & -2 \\ -4 & -1 & 0 & -1 & -4 \end{bmatrix}\)

This is a reflection across the \(x\)-axis and a reflection across the \(y\)-axis. However, because the graph is symmetric with respect to the \(y\)-axis, a reflection across the \(y\)-axis does not change the graph.

\[
\begin{bmatrix}
2 & -1 & 0 & -2 \\
-4 & -1 & 0 & -2 \\
3 & 3 & 3 & 3 \\
7 & 4 & 3 & 4 \\
\end{bmatrix}
\]

\[20. \text{ a. Let} \ x \text{ represent the year and} \ y \text{ represent mean household population. Sample answer using} \ \text{the calculator's median-median command:} \]
\[
\hat{y} = -0.0247x + 51.71.
\]

\[\text{b. Sample answer: What was the mean household population in 1965? Substitute 1965 for} \ x: \]
\[
\hat{y} = -0.0247(1965) + 51.71 = 3.1745. \text{ In 1965, the mean household population was approximately 3.17.}
\]

c. \text{Sample answer: In what year will the mean household population be 2.00? Substitute 2.00 for} \ \hat{y}: \]
\[
2.00 = -0.0247x + 51.71, \text{ so} \ 0.0247x = 49.71, \\
\text{and} \ x = \frac{49.71}{0.0247} \approx 2013. \text{ The mean household population will be 2.00 in the year 2013.}
\]

**TAKE ANOTHER LOOK**

1. When \(\det = 1\), \[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix}^{-1} = \begin{bmatrix}
d & -b \\
-c & a \\
\end{bmatrix}.
\]

When \(\det = 2\), \[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix}^{-1} = \begin{bmatrix}
\frac{d}{2} & \frac{-b}{2} \\
\frac{-c}{2} & \frac{a}{2} \\
\end{bmatrix}.
\]

Conjecture: \[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix}^{-1} = \frac{1}{\det} \begin{bmatrix}
d & -b \\
-c & a \\
\end{bmatrix}.
\]

If a matrix is thought of as a transformation, the determinant gives the area of the image of the unit square, thus capturing the amount of stretching or shrinking. That's why the inverse matrix can be expressed with the determinant being divided.

2. Yes, it is a 90° counterclockwise rotation of all points and figures in the plane.

For other rotations:
\[
\begin{bmatrix}
-1 & 0 \\
0 & -1 \\
\end{bmatrix} \text{ produces a 180° rotation, and}
\]
\[
\begin{bmatrix}
0 & 1 \\
-1 & 0 \\
\end{bmatrix} \text{ produces a 90° clockwise rotation.}
\]

If you recall the exploration from Chapter 4 (or a theorem from geometry), you may think of these rotations as compositions of reflections.

For example, \[
\begin{bmatrix}
0 & -1 \\
1 & 0 \\
\end{bmatrix} \text{ interchanges} \ x \text{ and} \ y
\]
(a reflection across the line \(y = x\) and then negates the first coordinate (a reflection across the \(y\)-axis).

3. \[
\begin{bmatrix}
-10 & 10 & 1 & -5 & 10 \\
\end{bmatrix}
\]

Vertices: \((2.8, 5.2), (4.437, 3.563), (2.461, -0.980), (-5.373, 6.626), (-4.182, 7.818), (-2.295, -1.244)\)

Graph the equation for any value of \(P\) (the second graph shows the circle with \(P = 25\)) and imagine increasing the value of \(P\), enlarging the circle, until the last point within the feasible region is hit. This will show that the vertex on the far right provides the maximum value.

The maximum value of \(P\) occurs at \((4.437, 3.563)\), where \(P = 43.878\).

4. The matrix \[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix} \begin{bmatrix} 1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

reduces to
\[
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix} \begin{bmatrix}
\frac{d}{ad - bc} & \frac{-b}{ad - bc} \\
\frac{-c}{ad - bc} & \frac{a}{ad - bc} \\
\end{bmatrix}
\]

which means the inverse is
\[
\frac{1}{ad - bc} \begin{bmatrix}
d & -b \\
-c & a \\
\end{bmatrix}
\]

Because the determinant is defined as \(\det = ad - bc\), this is the same result as in Take Another Look activity 1.